JHEP HYPER VERSION

RECEIVED: *March 7, 2001*, ACCEPTED: *April 17, 2001*

Compact and noncompact gauged maximal supergravities in three dimensions

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ABSTRACT: We present the maximally supersymmetric three-dimensional gauged supergravities. Owing to the special properties of three dimensions — especially the on-shell duality between vector and scalar fields, and the purely topological character of (super)gravity — they exhibit an even richer structure than the gauged supergravities in higher dimensions. The allowed gauge groups are subgroups of the global $E_{8(8)}$ symmetry of ungauged $N = 16$ supergravity. They include the regular series $\mathrm{SO}(p, 8-p) \times \mathrm{SO}(p, 8-p)$ for all $p = 0, 1, \ldots, 4$, the group $E_{8(8)}$ itself, as well as various noncompact forms of the exceptional groups E_7, E_6 and $F_4 \times G_2$. We show that all these theories admit maximally supersymmetric ground states, and determine their background isometries, which are superextensions of the anti-de Sitter group $\mathrm{SO}(2, 2)$. The very existence of these theories is argued to point to a new supergravity beyond the standard $D = 11$ supergravity.

KEYWORDS: Chern-Simons Theories, Gauge Symmetry, Extended Supersymmetry, Supergravity Models.

*Supported in part by the European Union under Contract No. HPRN-CT-2000-00122 and No. HPRN-CT-2000-00131.

†UMR 8549: Unité Mixte du Centre National de la Recherche Scientifique, et de l'École Normale Supérieure.

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1. Introduction

In this article we explain in detail the construction of maximal gauged supergravities in three dimensions, recently announced in [1]. While maximal gauged supergravities in higher dimensions have been known for a long time, starting with the gauged $N = 8$ theory in four dimensions [2], and subsequently for dimensions $5 \leq D \leq 8$ [3]–[7], the results on gauged supergravities in three dimensions and below have remained somewhat fragmentary until now. The results presented in this paper close this gap. In addition they open up new perspectives: unlike maximal gauged supergravities in higher dimensions, the maximal AdS_3 supergravities, which we obtain here, are neither contained in nor derivable by any known mechanism from the known maximal supergravities in higher dimensions. The new, and purely field theoretic, evidence for a theory beyond $D = 11$ supergravity [8] and type-IIB supergravity [9, 10] that we have thus obtained is perhaps the most important consequence of the present work.

Topological gauged supergravities in three dimensions were first constructed in [11]; these theories are supersymmetric extensions of Chern-Simons (CS) theories with (n_L, n_R) supersymmetry and gauge group $\mathrm{SO}(n_L) \times \mathrm{SO}(n_R)$, but have no propagating matter degrees of freedom (see also [12] for earlier work on $D = 3$ supergravity). Matter coupled gauged supergravities can, of course, be obtained by direct dimensional reduction of gauged supergravities in $D \geq 4$ to three dimensions and below, but these do not preserve the maximal supersymmetry [13]. Another matter coupled theory with half maximal supersymmetry, obtained by compactifying the ten-dimensional $N = 1$ supergravity on a seven-sphere, has been discussed in [14] (however, [14] deals only with the bosonic part of the lagrangian). In a different vein, [15] constructs an abelian gauged supergravity by deforming the $D = 3$, $N = 2$ supergravity whose matter sector is described by an $\mathrm{SO}(n, 2)/\,\mathrm{SO}(n) \times \mathrm{SO}(2)$ coset space sigma model. This model bears some resemblance to the present construction in that the vector fields appear via a CS term rather than a Yang-Mills term, unlike the matter-coupled theories mentioned before. However, the construction is limited to the abelian case, whereas the present construction yields non-abelian CS theories, thereby providing the first examples of a non-abelian duality between scalars and vector fields in three space-time dimensions.

Gauged supergravities have attracted strong interest again recently in the context of the conjectured duality between AdS supergravities and superconformal quantum field theories on the AdS boundary [16]. For instance, classical supergravity domain wall solutions are claimed to encode the information on the renormalization group flow of the strongly coupled gauge theory [17]. The theories admitting AdS_3 ground states are expected to be of particular interest for the AdS/CFT duality due to the rich and rather well understood structure of two-dimensional superconformal field theories. However, a large part of the recent work dealing with the conjectured AdS/CFT correspondence in AdS_3 has been based on the BTZ black hole solution of [18], which has no propagating matter degrees of freedom in the bulk. We will see that the gauged $N = 16$ theories yield a rich variety of supersymmetric groundstates, virtually exhausting all the possible vacuum symmetries of AdS type listed in [19], and thus an equally rich variety of superconformal theories on the boundary.

As is well known [20], the scalar fields in the toroidal compactification of $D = 11$ supergravity [8] on a d-torus form a coset space sigma model manifold G/H with the exceptional group $G = E_{d(d)}$ and H its maximally compact subgroup; in particular, for $d = 8$ one obtains a theory with global $E_{8(8)}$ symmetry and local $SO(16)$ [21, 22].

The complete list of ungauged matter coupled supergravities in three dimensions (which unlike topological supergravities only exist with $N \leq 16$ supersymmetries) has been presented in [23]. Gauging any of these theories corresponds to promoting a subgroup G_0 of the rigid G symmetry group to a local symmetry in such a way that the full local supersymmetry is preserved. The latter requirement engenders additional Yukawa-like couplings between the scalars and fermions, as well as a rather complicated potential for the scalar fields. As we will demonstrate by explicit construction, the possible compact and non-compact non-abelian gauge groups, all of which are subgroups of the global $E_{8(8)}$ symmetry of the ungauged maximal supergravity theory and preserve the full local $N = 16$ supersymmetry, are more numerous in three dimensions than in higher dimensions.

There are essentially two properties which distinguish the three dimensional models from all their higher dimensional relatives. First, the gravitational sector does not contain any propagating degrees of freedom such that the theories without matter coupling may be formulated as CS theories of AdS supergroups [11]; see also the classic article [24] for a description of the peculiarities of gravity in three space-time dimensions. In fact, pure quantum gravity [25, 26] and quantum supergravity [27] are exactly solvable in three space-time dimensions. Second, in three dimensions scalar fields are on-shell equivalent to vector fields. At the linearized level, this duality is encapsulated in the relation

$$\epsilon_{\mu\nu\rho}\, \partial^\rho \varphi^m = \partial_{[\mu} B_{\nu]}{}^m \, . \tag{1.1}$$

This relation plays a special role in the derivation of maximal $N = 16$ supergravity in three dimensions [21, 22, 28, 29]: in order to expose its rigid $E_{8(8)}$ symmetry, all vector fields obtained by dimensional reduction of $D = 11$ supergravity [8] on an 8-torus must be dualized into scalar fields. Vice versa, the duality (1.1) allows us to redualize part of the scalar fields into vector fields, such that the ungauged theory possesses different equivalent formulations which are related by duality [28].

As explained there, the replacement of scalar fields by vector fields breaks the exceptional $E_{8(8)}$ symmetry; when attempting to gauge this theory while maintaining its $E_{8(8)}$ structure and thus keeping all the scalars, it is therefore a priori not clear how to re-incorporate the vector fields necessary for the gauging without introducing new and unwanted propagating degrees of freedom. We will circumvent this apparent problem by interpreting (1.1) as defining up to 248 vector fields as (nonlocal) functions of the scalar fields. This freedom in the choice of the number of vector fields is at the origin of the large number of possible gauge groups that we encounter in three dimensions.

In higher dimensions, the gauge group is to a large extent determined by the number and transformation behavior of the vector fields under the rigid G symmetry of the ungauged theory. As a necessary condition for gauging a subgroup $G_0 \subset G$, the vector fields or at least a maximal subset thereof must transform in the adjoint representation of G_0. In the latter case there may remain additional vector fields which transform nontrivially under the gauge group. Upon gauging, these charged vector fields would acquire mass terms and thereby spoil the matching of bosonic and fermionic degrees of freedom; to avoid such inconsistencies one needs some additional mechanism to accommodate these degrees of freedom. Altogether, this does not leave much freedom for the choice of the gauge group. In $D = 4$ and $D = 7$ one must make use of the full set of vector fields transforming in the adjoint representation of the gauge groups SO(8) and SO(5), respectively. The situation is more subtle in dimensions $D = 5, 6$ where only a subset of the vector fields transforms in the adjoint representation of the gauge groups SO(6) and SO(5), respectively. The problem of coupling charged vector fields is circumvented in $D = 5$ by dualizing the additional vector fields into massive self-dual two forms [4, 6]; in $D = 6$ they are absorbed by massive gauge transformations of the two forms [7].

By contrast the proper choice of gauge group is much less obvious in three dimensions. With (1.1), we may introduce for any subgroup $G_0 \subset E_{8(8)}$ a set of $\nu = \dim G_0$ vector fields transforming in the adjoint representation of G_0. A priori, there is no restriction on the choice of G_0; however, demanding maximal supersymmetry of the gauged theory strongly restricts the possible choices for G_0. It is one of our main results that the entire set of consistency conditions for the three-dimensional gauged theory may be encoded into a single algebraic condition

$$\mathbb{P}_{\mathbf{27000}}\, \Theta = 0\,, \tag{1.2}$$

where Θ is the embedding tensor characterizing the subgroup G_0, and $\mathbb{P}$ a projector in the $E_{8(8)}$ tensor product decomposition $(\mathbf{248} \times \mathbf{248})_{\rm sym} = \mathbf{1} + \mathbf{3875} + \mathbf{27000}$. Solutions to (1.2) may be constructed by purely group theoretical considerations. Having formulated the consistency conditions of the gauged theory as a projector condition for the embedding tensor of the gauge group allows us to construct a variety of models with maximal local supersymmetry. As a result, we identify a "regular" series of gauged theories with gauge group $\mathrm{SO}(p, 8-p) \times \mathrm{SO}(p, 8-p)$, including the maximal compact gauge group $\mathrm{SO}(8) \times \mathrm{SO}(8)$ as a special case. In addition, we find several theories with exceptional noncompact gauge groups, among them an extremal theory which gauges the full $E_{8(8)}$ symmetry. These theories have no analog in higher dimensions.

This collection of maximal admissible gauge groups is presented in table 1; all the gauge groups — apart from the theory with local $E_{8(8)}$ — have two simple factors with a fixed ratio of coupling constants. As a by-product of our construction we can understand and re-state the corresponding consistency conditions for the higher dimensional gauged supergravities of [2, 6] in very simple terms; in particular, the derivation of the T-identities for the $D = 4, 5$ theories can now be simplified considerably by reducing it to purely group theoretical condition analogous to (1.2). Remarkably, and even though the rigid $G = E_{d(d)}$ symmetry of the ungauged theory

gauge group G_0	ratio of coupling constants
$\mathrm{SO}(p,8-p)\times\mathrm{SO}(p,8-p)$	$g_1/g_2=-1$
$G_{2(2)}\times F_{4(4)}$	$g_{G_2}/g_{F_4}=-3/2$
$G_2\times F_{4(-20)}$	
$E_{6(6)}\times\mathrm{SL}(3)$	$g_{A_2}/g_{E_6}=-2$
$E_{6(2)}\times\mathrm{SU}(2,1)$	
$E_{6(-14)}\times\mathrm{SU}(3)$	
$E_{7(7)}\times\mathrm{SL}(2)$	$g_{A_1}/g_{E_7}=-3$
$E_{7(-5)}\times\mathrm{SU}(2)$	
$E_{8(8)}$	g_{E_8}

Table 1: Regular and exceptional admissible gauge groups.

is broken, the construction and proof of consistency of the gauged theory makes essential use of the properties of the maximal symmetry group $E_{d(d)}$ in all cases.

This paper is organized as follows. In section 2 we review the ungauged $N=16$ theory and in particular discuss the full nonlinear version of the duality (1.1) between scalar and vector fields. In section 3 we present the lagrangian of the gauged theory. It is characterized by a set of tensors $A_{1,2,3}$ which are nonlinear functions of the scalar fields and describe the Yukawa-type couplings between fermions and scalars as well as the scalar potential. We derive the consistency conditions that these tensors must satisfy in order for the full $N=16$ supersymmetry to be preserved, and show that $A_{1,2,3}$ combine into a "T-tensor" analogous to the one introduced in [2], but now transforming as the $\mathbf{1}+\mathbf{3875}$ of $E_{8(8)}$. In section 4 we show that these consistency conditions imply and may entirely be encoded into the algebraic equation (1.2) for the embedding tensor of the gauge group, which selects the admissible gauge groups $G_0\subset E_{8(8)}$. In turn, every solution to (1.2) yields a nontrivial solution for $A_{1,2,3}$ in terms of the scalar fields which satisfies the full set of consistency conditions. Maximal supersymmetry of the gauged theory thus translates into a simple projector equation for the gauge group G_0.

In section 5 we analyze equation (1.2) and its solutions among the maximal subgroups of SO(16) and $E_{8(8)}$, respectively. We find the maximal compact admissible gauge group $G_0=\mathrm{SO}(8)\times\mathrm{SO}(8)$ as well as its noncompact real forms $\mathrm{SO}(p,8-p)\times\mathrm{SO}(p,8-p)$ for $p=1,\ldots,4$. In addition, we identify the exceptional noncompact gauge groups given in table 1. Each of these groups gives rise to a maximally supersymmetric gauged supergravity. Section 6 is devoted to an analysis of stationary points of the scalar potential which preserve the maximal number of 16 supersymmetries. We show that all our theories admit a maximally symmetric ground state and determine their background isometries. Finally we speculate on a possible higher dimensional origin of these theories.

2. The ungauged $N = 16$ theory

We first summarize the pertinent results about (ungauged) maximal $N = 16$ supergravity in three dimensions. The complete lagrangian and supersymmetry transformations were presented in [22], whose conventions and notation we follow throughout this paper.[1] The physical fields of $N = 16$ supergravity constitute an irreducible supermultiplet with 128 bosons and 128 fermions transforming as inequivalent fundamental spinors of SO(16). In addition, the theory contains the dreibein $e_\mu{}^\alpha$ and 16 gravitino fields ψ^I_μ, which do not carry propagating degrees of freedom in three dimensions. As first shown in [21], it possesses a "hidden" invariance under rigid $E_{8(8)}$ and local SO(16) transformations. Consequently, the scalar fields are described by an element $\mathcal{V}$ of the non-compact coset space $E_{8(8)}/\,\mathrm{SO}(16)$ in the fundamental 248-dimensional representation of $E_{8(8)}$, which transforms as

$$\mathcal{V}(x) \longrightarrow g\,\mathcal{V}(x)\,h^{-1}(x)\,, \qquad g \in E_{8(8)}\,,\ h(x) \in \mathrm{SO}(16)\,, \tag{2.1}$$

(see appendix A for our $E_{8(8)}$ conventions). The scalar fields couple to the fermions via the currents

$$\mathcal{V}^{-1}\partial_\mu\mathcal{V} = \frac{1}{2}Q^{IJ}_\mu X^{IJ} + P^A_\mu Y^A\,. \tag{2.2}$$

The composite SO(16) connection Q^{IJ}_μ enters the covariant derivative D_μ in

$$\begin{aligned} D_\mu\psi^I_\nu &:= \partial_\mu\psi^I_\nu + \frac{1}{4}\,\omega_\mu{}^{ab}\,\gamma_{ab}\,\psi^I_\nu + Q^{IJ}_\mu\psi^J_\nu\,, \\ D_\mu\chi^{\dot A} &:= \partial_\mu\chi^{\dot A} + \frac{1}{4}\,\omega_\mu{}^{ab}\,\gamma_{ab}\,\chi^{\dot A} + \frac{1}{4}\,Q^{IJ}_\mu\Gamma^{IJ}_{\dot A\dot B}\,\chi^{\dot B}\,. \end{aligned} \tag{2.3}$$

Definition (2.2) implies the integrability relations:

$$Q^{IJ}_{\mu\nu} + \frac{1}{2}\,\Gamma^{IJ}_{AB}\,P^A_\mu P^B_\nu = 0\,, \qquad D_{[\mu}P^A_{\nu]} = 0\,, \tag{2.4}$$

where the SO(16) field strength is defined as

$$Q^{IJ}_{\mu\nu} := \partial_\mu Q^{IJ}_\nu - \partial_\nu Q^{IJ}_\mu + 2\,Q^{K[I}_\mu Q^{J]K}_\nu\,.$$

The full supersymmetry variations read [22]

$$\begin{aligned} \delta e_\mu{}^\alpha &= \mathrm{i}\overline{\epsilon}^I\gamma^\alpha\psi^I_\mu\,, & \delta\,\psi^I_\mu &= D_\mu\epsilon^I - \frac{1}{4}\mathrm{i}\gamma^\nu\epsilon^J\,\overline{\chi}\,\Gamma^{IJ}\gamma_{\mu\nu}\chi\,, \\ \mathcal{V}^{-1}\delta\mathcal{V} &= \Gamma^I_{A\dot A}\,\overline{\chi}^{\dot A}\epsilon^I Y^A\,, & \delta\,\chi^{\dot A} &= \frac{\mathrm{i}}{2}\,\gamma^\mu\epsilon^I\,\Gamma^I_{A\dot A}\,\widehat{P}^A_\mu\,, \end{aligned} \tag{2.5}$$

with the supercovariant current

$$\widehat{P}^A_\mu := P^A_\mu - \overline{\psi}^I_\mu\chi^{\dot A}\Gamma^I_{A\dot A}\,.$$

[1] In particular we use the metric with signature $(+--)$ and three-dimensional gamma matrices with $e\,\gamma^{\mu\nu\rho} = -\mathrm{i}\epsilon^{\mu\nu\rho}$, where $\epsilon^{012} = \epsilon_{012} = 1$, and $e \equiv \det e_\mu{}^\alpha$ is the dreibein determinant.

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As shown in [22], they leave invariant the lagrangian [2]

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}eR + \frac{1}{4}eP^{\mu A}P_\mu^A + \frac{1}{2}\,\epsilon^{\lambda\mu\nu}\overline{\psi}_\lambda^I D_\mu \psi_\nu^I - \\ & -\frac{\mathrm{i}}{2}e\overline{\chi}^{\dot{A}}\gamma^\mu D_\mu \chi^{\dot{A}} - \frac{1}{2}e\,\overline{\chi}^{\dot{A}}\gamma^\mu\gamma^\nu\psi_\mu^I\,\Gamma^I_{A\dot{A}}P_\nu^A - \\ & -\frac{1}{8}e\Big(\overline{\chi}\gamma_\rho\Gamma^{IJ}\chi\,(\overline{\psi}_\mu^I\gamma^{\mu\nu\rho}\psi_\nu^J - \overline{\psi}_\mu^I\gamma^\rho\psi^{\mu J}) + \overline{\chi}\chi\,\overline{\psi}_\mu^I\gamma^\nu\gamma^\mu\psi_\nu^I\Big) + \\ & +e\Big(\frac{1}{8}(\overline{\chi}\chi)(\overline{\chi}\chi) - \frac{1}{96}\overline{\chi}\gamma^\mu\Gamma^{IJ}\chi\,\overline{\chi}\gamma_\mu\Gamma^{IJ}\chi\Big)\,. \end{aligned} \tag{2.6}$$

The invariance is most conveniently checked in 1.5 order formalism, with the torsion

$$T_{\mu\nu}{}^\rho = \frac{1}{2}\mathrm{i}\overline{\psi}_\mu^K\gamma^\rho\psi_\nu^K + \frac{1}{4}\mathrm{i}\,\overline{\chi}^{\dot{A}}\,\gamma_{\mu\nu}{}^\rho\chi^{\dot{A}}\,. \tag{2.7}$$

A central role in our construction is played by the on-shell duality between scalar fields and vector fields in three dimensions, which we shall now discuss. The scalar field equation induced by (2.6) is given by

$$D_\mu\left(e\,(P^{\mu A} - \overline{\psi}_\nu^I\gamma^\mu\gamma^\nu\chi^{\dot{A}}\Gamma^I_{A\dot{A}})\right) = \frac{1}{2}\epsilon^{\mu\nu\rho}\overline{\psi}_\mu^I\psi_\nu^J\Gamma^{IJ}_{AB}P_\rho^B + \frac{1}{8}\mathrm{i}e\,\overline{\chi}\gamma^\mu\Gamma^{IJ}\chi\,\Gamma^{IJ}_{AB}P_\mu^B\,. \tag{2.8}$$

Upon use of the Rarita-Schwinger and Dirac equations for ψ_μ^I and $\chi^{\dot{A}}$, respectively, this equation may be rewritten in the form

$$\partial^\mu\left(e\,\mathcal{J}_\mu{}^{\mathcal{M}}\right) = 0\,, \tag{2.9}$$

where $\mathcal{J}_\mu{}^{\mathcal{M}}$ is the conserved Noether current associated with the rigid $E_{8(8)}$ symmetry [31]:

$$\begin{aligned}\mathcal{J}^{\mu\mathcal{M}} = & \,2\mathcal{V}^{\mathcal{M}}{}_B\widehat{P}^{\mu B} - \frac{\mathrm{i}}{2}\mathcal{V}^{\mathcal{M}}{}_{IJ}\,\overline{\chi}\gamma^\mu\Gamma^{IJ}\chi - \\ & -2e^{-1}\epsilon^{\mu\nu\rho}\left(\mathcal{V}^{\mathcal{M}}{}_{IJ}\,\overline{\psi}_\nu^I\psi_\rho^J - \mathrm{i}\,\Gamma^I_{A\dot{A}}\mathcal{V}^{\mathcal{M}}{}_A\,\overline{\psi}_\nu^I\gamma_\rho\chi^{\dot{A}}\right). \end{aligned} \tag{2.10}$$

In writing this expression we have made use of the equivalence of the fundamental and adjoint representations of $E_{8(8)}$ which yields the relation (see also appendix A)

$$\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}} := \frac{1}{60}\,\mathrm{Tr}(t^{\mathcal{M}}\mathcal{V}t_{\mathcal{A}}\mathcal{V}^{-1})\,.$$

The existence of the conserved current (2.10) allows us to introduce 248 abelian vector fields $B_\mu{}^{\mathcal{M}}$ (with index $\mathcal{M} = 1, \ldots, 248$), via

$$\epsilon^{\mu\nu\rho}\,B_{\nu\rho}{}^{\mathcal{M}} = e\mathcal{J}^{\mu\mathcal{M}}\,, \tag{2.11}$$

[2]Note that the factor in front of the last term $(\overline{\chi}\gamma_\mu\Gamma^{IJ}\chi)^2$ differs from the one given in [22] as was already noticed in [30].

where $B_{\mu\nu}{}^{\mathcal{M}} := \partial_\mu B_\nu{}^{\mathcal{M}} - \partial_\nu B_\mu{}^{\mathcal{M}}$ denotes the abelian field strength. This equation defines the vector fields up to the $[\mathrm{U}(1)]^{248}$ gauge transformations

$$B_\mu{}^{\mathcal{M}} \to B_\mu{}^{\mathcal{M}} + \partial_\mu \Lambda^{\mathcal{M}} \,. \tag{2.12}$$

In accordance with (2.1) these vector fields transform in the adjoint representation of rigid $E_{8(8)}$ and are singlets under local SO(16). The supersymmetry transformations of the vector fields have not been given previously; they follow by "$E_{8(8)}$ covariantization" of the supersymmetry variations of the 36 vector fields obtained by direct dimensional reduction of $D = 11$ supergravity to three dimensions [32]

$$\delta B_\mu{}^{\mathcal{M}} = -2\, \mathcal{V}^{\mathcal{M}}{}_{IJ}\, \overline{\epsilon}^I \psi_\mu^J + \mathrm{i}\Gamma^I_{A\dot{A}}\, \mathcal{V}^{\mathcal{M}}{}_A\, \overline{\epsilon}^I \gamma_\mu \chi^{\dot{A}} \,. \tag{2.13}$$

This transformation must be compatible with the duality relation (2.11). To check this, it is convenient to rewrite the latter in terms of the supercovariant field strength

$$\widehat{B}_{\mu\nu}{}^{\mathcal{M}} := B_{\mu\nu}{}^{\mathcal{M}} + 2\, \mathcal{V}^{\mathcal{M}}{}_{IJ}\, \overline{\psi}^I_\mu \psi^J_\nu - 2\mathrm{i}\, \Gamma^I_{A\dot{A}} \mathcal{V}^{\mathcal{M}}{}_A\, \overline{\psi}^I_{[\mu} \gamma_{\nu]} \chi^{\dot{A}} \,,$$

whose supercovariance is straightforwardly verified from (2.13). The duality relation (2.11) then takes the following supercovariant form

$$\epsilon^{\mu\nu\rho} \widehat{B}_{\nu\rho}{}^{\mathcal{M}} = 2e\, \mathcal{V}^{\mathcal{M}}{}_A \widehat{P}^{\mu A} - \frac{\mathrm{i}}{2}\, e \mathcal{V}^{\mathcal{M}}{}_{IJ}\, \overline{\chi} \gamma^\mu \Gamma^{IJ} \chi \,. \tag{2.14}$$

Equation (2.14) consistently defines the dual vector fields as nonlocal and nonlinear functions of the original 248 scalar fields (including the 120 gauge degrees of freedom associated with local SO(16)), provided the latter obey their equations of motion. We emphasize that in this way we can actually introduce as many vector fields as there are scalar fields, whereas the direct dimensional reduction of $D = 11$ supergravity to three dimensions produces only 36 vector fields. The "$E_{8(8)}$ covariantization" alluded to above simply consists in extending the relevant formulas from these 36 vectors to the full set of $\dim G_0 \leq 248$ vector fields in a way that respects the $E_{8(8)}$ structure of the theory. In the ungauged theory the vector fields have been introduced merely on-shell; there is no lagrangian formulation that would comprise the scalar fields as well as their dual vector fields. However, we shall see that the gauged theory provides a natural off-shell framework which accommodates both the scalars and their dual vectors.

From (2.14) we can also extract the equation of motion of the dual vectors: acting on both sides with $\epsilon_{\rho\mu\nu}\partial^\nu$ and making use of the integrability relations (2.4), we obtain

$$\partial_\nu B^{\mu\nu\mathcal{M}} = -\frac{1}{2}\, e^{-1}\, \epsilon^{\mu\nu\rho}\, \mathcal{V}^{\mathcal{M}}{}_{IJ}\, Q^{IJ}_{\nu\rho} + \text{fermionic terms} \,. \tag{2.15}$$

Also the fermionic terms still depend on the original scalar fields. This is obvious from the fact that we need the scalar field matrix $\mathcal{V}$ to convert the SO(16) indices on the fermions into the $E_{8(8)}$ indices appropriate for the l.h.s. of this equation. (Let us note already here that in the gauged theory, the r.h.s. of this equation will acquire additional contributions containing $B_{\mu\nu}{}^{\mathcal{M}}$ in order of the coupling constant). We recognize an important difference between the "dual formulations" of the theory: whereas the vectors disappear completely in the standard formulation of the theory, the vector equations of motion in general still depend on the dual scalar fields. It is only under very special circumstances, and for special subsets of the 248 vector fields, that one can completely eliminate the associated dual scalars. This is obviously the case for the version obtained by direct reduction of $D = 11$ supergravity to three dimensions where only 92 bosonic degrees of freedom appear as scalar fields while 36 physical degrees of freedom appear as vector fields. As shown in [28], the latter are associated with the 36-dimensional maximal nilpotent commuting subalgebra of $E_{8(8)}$, but there are further intermediate possibilities.

To conclude this section, we recall that the three dimensional Einstein-Hilbert term can be rewritten in Chern-Simons form as

$$-\frac{1}{4}eR = \frac{1}{4}\,\epsilon^{\mu\nu\rho}\,e_\mu{}^a\,F_{\nu\rho\,a}\,, \tag{2.16}$$

by means of the dual spin connection

$$A^a_\mu = -\frac{1}{2}\epsilon^{abc}\omega_{\mu\,bc}\,,$$

with field strength $F^a_{\mu\nu} = 2\partial_{[\mu}A^a_{\nu]} + \epsilon^a{}_{bc}\,A^b_\mu A^c_\nu$. When gauging the theory the Minkowski background space-time will be deformed to an AdS_3 spacetime characterized by

$$R_{\mu\nu} = 2m^2 g_{\mu\nu}\,, \tag{2.17}$$

with (negative) cosmological constant $\Lambda = -2m^2$. The Lorentz-covariant derivative is accordingly modified to an AdS_3 covariant derivative

$$\mathcal{D}^\pm_\mu := \partial_\mu + \frac{1}{2}\mathrm{i}\gamma_a(A_\mu{}^a \pm m e_\mu{}^a)\,, \tag{2.18}$$

with commutator

$$[\mathcal{D}^\pm_\mu, \mathcal{D}^\pm_\nu] = \frac{1}{2}\mathrm{i}\gamma_a(F_{\mu\nu}{}^a + m^2\epsilon^{abc}e_{\mu b}e_{\nu c})\,.$$

We will return to these formulas when discussing the conditions for (n_L, n_R) supersymmetry in AdS_3 in section 6.

3. Gauged $N = 16$ supergravity

The lagrangian (2.6) is invariant under rigid $E_{8(8)}$ and local SO(16). To gauge the theory, we now select a subgroup $G_0 \subset E_{8(8)}$ which will be promoted to a local symmetry. The resulting theory will then be invariant under local $G_0 \times SO(16)$, such that (2.1) is replaced by

$$\mathcal{V}(x) \longrightarrow g_0(x)\,\mathcal{V}(x)\,h^{-1}(x)\,, \qquad g_0(x) \in G_0\,,\ h(x) \in \mathrm{SO}(16)\,, \tag{3.1}$$

However, it should be kept in mind that the local symmetries are realized in different ways: as before, the local SO(16) is realized in terms of "composite" gauge connections, whereas the gauge fields associated with the local G_0 symmetry are independent fields to begin with. Restricting to semisimple subgroups, G_0 is properly characterized by means of its embedding tensor $\Theta_{\mathcal{M}\mathcal{N}}$ which is the restriction of the Cartan-Killing form $\eta_{\mathcal{M}\mathcal{N}}$ onto the associated algebra $\mathfrak{g}_0$. The embedding tensor will have the form

$$\Theta_{\mathcal{M}\mathcal{N}} = \sum_j \varepsilon_j\, \eta^{(j)}_{\mathcal{M}\mathcal{N}}\,, \tag{3.2}$$

where $\eta^{\mathcal{M}\mathcal{N}}\eta^{(j)}_{\mathcal{N}\mathcal{K}}$ project onto the simple subfactors of G_0, and the numbers ε_j correspond to the relative coupling strengths. It will turn out that these coefficients are completely fixed by group theory, so there is only one overall gauge coupling constant g. Owing to the symmetry of the projectors $\eta^{(j)}$ the embedding tensor is always symmetric:

$$\Theta_{\mathcal{M}\mathcal{N}} = \Theta_{\mathcal{N}\mathcal{M}}\,. \tag{3.3}$$

As discussed in the introduction we introduce a subset of $\nu = \dim G_0$ vector fields, obtained from (2.14) by projection with $\Theta_{\mathcal{M}\mathcal{N}}$. For these we introduce special labels $m, n, \ldots$, with the short hand notation

$$B_\mu{}^m t_m \equiv B_\mu{}^{\mathcal{M}}\,\Theta_{\mathcal{M}\mathcal{N}}\,t^{\mathcal{N}}\,, \qquad \text{etc.} \tag{3.4}$$

Note that we do not make any assumption about G_0 at this point; in particular, our ansatz allows for compact as well as noncompact gauge groups. The possible choices for G_0 will be determined in section 5. The first step is the covariantization of derivatives in (2.2) according to

$$\mathcal{V}^{-1}\mathcal{D}_\mu\mathcal{V} \equiv \mathcal{V}^{-1}\partial_\mu\mathcal{V} + g\,B_\mu{}^m\,\mathcal{V}^{-1}t_m\mathcal{V} \equiv \mathcal{P}^A_\mu Y^A + \frac{1}{2}\mathcal{Q}^{IJ}_\mu X^{IJ}\,, \tag{3.5}$$

with gauge coupling constant g. The non-abelian field strength reads

$$\mathcal{B}_{\mu\nu}{}^m := \partial_\mu\,B_\nu{}^m - \partial_\nu\,B_\mu{}^m + g\,f^m{}_{np}\,B_\mu{}^n B_\nu{}^p\,. \tag{3.6}$$

The integrability relations (2.4) are modified to

$$\begin{aligned} \mathcal{Q}^{IJ}_{\mu\nu} + \frac{1}{2}\,\Gamma^{IJ}_{AB}\,\mathcal{P}^A_\mu\mathcal{P}^B_\nu &= g\,\mathcal{B}_{\mu\nu}{}^m\,\Theta_{mn}\mathcal{V}^n{}_{IJ}\,, \\ 2D_{[\mu}\mathcal{P}^A_{\nu]} &= g\,\mathcal{B}_{\mu\nu}{}^m\,\Theta_{mn}\mathcal{V}^n{}_A\,. \end{aligned} \tag{3.7}$$

With the hidden g dependent extra terms in the definition of the currents in (3.5), their supersymmetry variations become

$$\begin{aligned}\delta \mathcal{Q}_\mu^{IJ} &= \frac{1}{2}(\Gamma^{IJ}\Gamma^K)_{A\dot{A}}\mathcal{P}_\mu^A\,\overline{\chi}^{\dot{A}}\epsilon^K + g(\delta B_\mu{}^m)\,\Theta_{mn}\mathcal{V}^n{}_{IJ}\,,\\ \delta \mathcal{P}_\mu^A &= \Gamma^I_{A\dot{A}}\,D_\mu(\overline{\chi}^{\dot{A}}\epsilon^I) + g(\delta B_\mu{}^m)\,\Theta_{mn}\mathcal{V}^n{}_A\,,\end{aligned} \tag{3.8}$$

with the variation of the vector fields given in (2.13).

Both modifications violate the supersymmetry of the original lagrangian. In order to restore local supersymmetry we follow the standard Noether procedure as in [2], modifying both the original lagrangian as well as the transformation rules by g-dependent terms. We will first state the results, and then explain their derivation and comment on the special and novel features of our construction.

The full lagrangian can be represented in the form

$$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(3)}\,, \tag{3.9}$$

where $\mathcal{L}^{(0)}$ is just the original lagrangian (2.6), but with the modified currents defined in (3.5); thus $\mathcal{L}^{(0)}$ and $\mathcal{L}$ differ by terms of order $\mathcal{O}(g)$. The contributions $\mathcal{L}^{(1)}$ and $\mathcal{L}^{(2)}$ are likewise of order g and describe the Chern-Simons coupling of the vector fields and the Yukawa type couplings between scalars and fermions, respectively:

$$\mathcal{L}^{(1)} = -\frac{1}{4}\,g\,\epsilon^{\mu\nu\rho}\,B_\mu{}^m\Big(\partial_\nu B_{\rho\,m} + \frac{1}{3}\,g f_{mnp}\,B_\nu{}^n B_\rho{}^p\Big)\,, \tag{3.10}$$

$$\mathcal{L}^{(2)} = \frac{1}{2}ge\,A_1^{IJ}\overline{\psi}_\mu^I\,\gamma^{\mu\nu}\,\psi_\nu^J + \mathrm{i}ge\,A_2^{I\dot{A}}\overline{\chi}^{\dot{A}}\,\gamma^\mu\,\psi_\mu^I + \frac{1}{2}ge\,A_3^{\dot{A}\dot{B}}\overline{\chi}^{\dot{A}}\,\chi^{\dot{B}}\,, \tag{3.11}$$

where the tensors $A_{1,2,3}$ are functions of the scalar matrix $\mathcal{V}$ which remain to be determined. At order $\mathcal{O}(g^2)$, there is the scalar field potential $W(\mathcal{V})$:

$$\mathcal{L}^{(3)} = eW \equiv \frac{1}{8}\,g^2\,e\,\Big(A_1^{IJ}A_1^{IJ} - \frac{1}{2}\,A_2^{I\dot{A}}A_2^{I\dot{A}}\Big)\,. \tag{3.12}$$

Besides the extra g dependent terms induced by the modified currents, the supersymmetry variations must be amended by the following $\mathcal{O}(g)$ terms:

$$\delta_g\psi_\mu^I = \mathrm{i}g\,A_1^{IJ}\,\gamma_\mu\epsilon^J\,, \qquad \delta_g\chi^{\dot{A}} = g\,A_2^{I\dot{A}}\,\epsilon^I\,. \tag{3.13}$$

Of course, the above modifications of the lagrangian and the supersymmetry transformation rules have not been guessed "out of the blue", but at this point simply constitute an ansatz that has been written down in analogy with known gauged supergravities, in particular the $N = 8$ theory of [2]. The consistency of this ansatz must now be established by explicit computation.

The SO(16) tensors $A_{1,2,3}$ depending on the scalar fields $\mathcal{V}$ introduce Yukawa-type couplings between the scalars and the fermions beyond the derivative couplings generated by (2.2), as well as a potential for the scalar fields. As is evident from their definition, the tensors A_1^{IJ} and $A_3^{\dot{A}\dot{B}}$ are symmetric in their respective indices. Therefore, A_1^{IJ} decomposes as $1+135$ under SO(16),[3] viz.

$$A_1^{IJ} = A_1^{(0)}\delta^{IJ} + \tilde{A}_1^{IJ}\,, \tag{3.14}$$

with $\tilde{A}^{JJ}=0$, while for $A_3^{\dot{A}\dot{B}}$ we have the decomposition

$$A_3^{\dot{A}\dot{B}} = A_3^{(0)}\delta^{\dot{A}\dot{B}} + \tilde{A}_3^{\dot{A}\dot{B}}\,, \tag{3.15}$$

where

$$\tilde{A}_3^{\dot{A}\dot{B}} = \frac{1}{4!}A^{(4)}_{3\,IJKL}\Gamma^{IJKL}_{\dot{A}\dot{B}} + \frac{1}{2\cdot 8!}A^{(8)}_{3\,I_1\ldots I_8}\Gamma^{I_1\ldots I_8}_{\dot{A}\dot{B}}\,.$$

Therefore A_3 can contain the representations $1+1820+6435$. However, we will see that the 6435 drops out. Due to the occurrence of the 1820 in this decomposition, the tensor A_3 cannot be expressed in terms of $A_{1,2}$ unlike for $D=4$ and $D=5$. The independence of A_3 is a new feature of the $D=3$ gauged theory.

Several restrictions on the tensors $A_{1,2,3}$ can already be derived by imposing closure of the supersymmetry algebra on various fields at order $\mathcal{O}(g)$. Computing the commutator on the dreibein field we obtain an extra Lorentz rotation with parameter

$$\Lambda_{\alpha\beta} = 2gA_1^{IJ}\,\bar{\epsilon}_1^I\gamma_{\alpha\beta}\epsilon_2^J\,, \tag{3.16}$$

while evaluation of the commutator on the vector fields and the scalar field matrix $\mathcal{V}$ yields an extra gauge transformation with parameter

$$\Lambda^m = 2\,\mathcal{V}^m{}_{IJ}\,\bar{\epsilon}_1^I\epsilon_2^J + \mathrm{i}B_\mu{}^m\,\bar{\epsilon}_1^I\gamma^\mu\epsilon_2^I\,. \tag{3.17}$$

The latter induces a further SO(16) rotation with parameter $\omega^{IJ} = g\Lambda_m\mathcal{V}^m{}_{IJ}$ on $\mathcal{V}$ (as well as the fermions which transform under SO(16)). For the derivation of this result we need the relations

$$\mathcal{V}^m{}_A\,\Gamma^{(I}_{A\dot{A}}\,A_2^{J)\dot{A}} = \mathcal{V}^m{}_{IK}\,A_1^{JK} + \mathcal{V}^m{}_{JK}\,A_1^{IK}\,, \tag{3.18}$$

$$\Gamma^{[I}_{A\dot{A}}\,A_2^{J]\dot{A}} = \mathcal{V}^{\mathcal{C}}{}_{IJ}\Theta_{\mathcal{C}\mathcal{D}}\mathcal{V}^{\mathcal{D}}{}_A\,, \tag{3.19}$$

which give the first restrictions on the tensors $A_{1,2,3}$. A peculiarity is that the closure of the superalgebra on $B_\mu{}^m$ requires use of the duality equation, whereas the equations of motion are not needed to check closure on the remaining bosonic fields.

Tracing (3.18) over the indices I and J and using the symmetry of A_1^{IJ} we immediately obtain

$$\Gamma^I_{A\dot{A}}A_2^{I\dot{A}} = 0\,. \tag{3.20}$$

The tensor $A_2^{I\dot{A}}$ thus transforms as the $\overline{1920}$ (traceless vector spinor) representation of SO(16).

[3] Here and in the following, representations of SO(16) are written with ordinary numbers, while representations of $E_{8(8)}$ are given in boldface numbers.

To state the restrictions imposed on these tensors by the requirement of local supersymmetry more concisely, we now define the T-tensor

$$T_{\mathcal{A}|\mathcal{B}} := \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}\mathcal{V}^{\mathcal{N}}{}_{\mathcal{B}}\,\Theta_{\mathcal{M}\mathcal{N}}\,. \tag{3.21}$$

Clearly $T_{\mathcal{A}|\mathcal{B}} = T_{\mathcal{B}|\mathcal{A}}$ by the symmetry of Θ. Unlike the cubic expressions in [2] and [6], however, the T-tensor is quadratic in $\mathcal{V}$ due to the equivalence of the fundamental and adjoint representations for $E_{8(8)}$, see (A.4). The tensors $A_{1,2,3}$ must be expressible in terms of T if the theory can be consistently gauged. The detailed properties of the T-tensor will be the subject of the following section.

Let us next consider the consistency conditions for local supersymmetry of (3.9) step by step. All cancellations that are G_0-covariantizations of the corresponding terms in the ungauged theory will work as before, and for this reason we need only discuss those variations which have no counterpart in the ungauged theory. Variation of $\mathcal{L}^{(1)}$ produces only the contribution

$$\delta\mathcal{L}^{(1)} = -\frac{1}{4}g\epsilon^{\mu\nu\rho}\delta B_\mu{}^m\mathcal{B}_{\nu\rho m}\,,$$

because the CS term depends on no other fields but $B_\mu{}^m$. Inserting (2.13) the above variation can be seen to cancel against the extra terms in the variation of $\mathcal{L}^{(0)}$ arising in the integrability conditions, cf. (3.7).

A second set of g-dependent terms is obtained by varying $B_\mu{}^m$ in $\mathcal{Q}_\mu$ and $\mathcal{P}_\mu$, cf. (3.8). Expressing the result by means of the T-tensor, we obtain

$$\begin{aligned} &g\left(2\,T_{IJ|KL}\,\overline{\epsilon}^I\psi^J_\mu - \mathrm{i}\,T_{KL|A}\,\Gamma^I_{A\dot{B}}\overline{\epsilon}^I\gamma_\mu\chi^{\dot{B}}\right)\left(\overline{\psi}{}^K_\nu\,\gamma^{\mu\nu\rho}\,\psi^L_\rho + \frac{\mathrm{i}}{4}\overline{\chi}\gamma^\mu\Gamma^{KL}\chi\right) - \\ &\quad - g\left(T_{A|KL}\,\overline{\epsilon}^K\psi^L_\mu - \frac{1}{2}\mathrm{i}\,T_{A|B}\,\Gamma^K_{B\dot{B}}\overline{\epsilon}^K\gamma_\mu\chi^{\dot{B}}\right)\left(\mathcal{P}^{\mu A} - \overline{\chi}^{\dot{A}}\gamma^\nu\gamma^\mu\psi^I_\nu\,\Gamma^I_{A\dot{A}}\right). \end{aligned}$$

These terms combine with the variations of the fermionic fields from $\mathcal{L}^{(2)}$ and the new variations (3.13) in $\mathcal{L}^{(0)}$. Consideration of the $\epsilon\psi\mathcal{P}$ and $\epsilon\chi\mathcal{P}$ terms now reproduces (3.19), but in addition requires the differential relations

$$\begin{aligned} \mathcal{D}_\mu A_1^{IJ} &= \mathcal{P}_\mu{}^A\,\Gamma^{(I}_{A\dot{A}}\,A_2^{J)\dot{A}}\,, \\ \mathcal{D}_\mu A_2^{I\dot{A}} &= \frac{1}{2}\mathcal{P}_\mu{}^A\left(\Gamma^I_{A\dot{B}}\,A_3^{\dot{A}\dot{B}} + \Gamma^J_{A\dot{A}}\,A_1^{IJ}\right) - \frac{1}{2}\mathcal{P}_\mu{}^A\,\Gamma^I_{B\dot{A}}\,T_{A|B}\,. \end{aligned} \tag{3.22}$$

Multiplying the second relation by $\Gamma^I_{A\dot{A}}$ and invoking (3.20) yields

$$T_{A|B} = (A_1^{(0)} + A_3^{(0)})\,\delta_{AB} + \frac{1}{16}\Gamma^I_{A\dot{A}}\,\tilde{A}_3^{\dot{A}\dot{B}}\,\Gamma^I_{\dot{B}B}\,. \tag{3.23}$$

Since $\Gamma^I\Gamma^{(8)}\Gamma^I = 0$ there is no $\overline{6435}$ of SO(16) in $T_{A|B}$. However, the argument does not yet suffice to rule out such a contribution in A_3.

As in [2], the supersymmetry variation of the tensors $A_{1,2}$ is obtained from (3.22) by replacing $\mathcal{P}_\mu^A$ by $\Gamma^I_{A\dot{A}}\overline{\epsilon}^I\chi^{\dot{A}}$:

$$\begin{aligned}
\delta\tilde{A}_1^{IJ} &= \Gamma^K_{A\dot{B}}\overline{\epsilon}^K\chi^{\dot{B}}\,\Gamma^{(I}_{A\dot{A}}\,A_2^{J)\dot{A}}\,, \\
\delta A_2^{I\dot{A}} &= \frac{1}{2}\,\Gamma^K_{A\dot{B}}\overline{\epsilon}^K\chi^{\dot{B}}\left(\Gamma^I_{A\dot{C}}\,A_3^{\dot{A}\dot{C}} + \Gamma^J_{A\dot{A}}\,A_1^{IJ} - \Gamma^I_{B\dot{A}}\,T_{A|B}\right).
\end{aligned} \tag{3.24}$$

The tracelessness of $A_2^{I\dot{A}}$ in (3.20) in conjunction with (3.22) also implies that $A_1^{(0)}$ and $A_3^{(0)}$ are constant. This is consistent with the fact that the trace parts drop out from the above variations. Observe that the supersymmetry variation of A_3 does not yet enter at this point as it appears only at cubic order in the fermions.

At $\mathcal{O}(g^2)$ we get two quadratic identities. The first multiplies the $g^2\psi\epsilon$ variations and is straightforwardly obtained

$$A_1^{IK}A_1^{KJ} - \frac{1}{2}\,A_2^{I\dot{A}}A_2^{J\dot{A}} = \frac{1}{16}\,\delta^{IJ}\left(A_1^{KL}A_1^{KL} - \frac{1}{2}\,A_2^{K\dot{A}}A_2^{K\dot{A}}\right). \tag{3.25}$$

The second comes from the $g^2\chi\epsilon$ variations: performing the $\mathcal{O}(g)$ variations in $\mathcal{L}^{(2)}$ we obtain

$$\delta_g\mathcal{L}^{(2)} = g^2 e\overline{\chi}^{\dot{A}}\epsilon^I\left(-3A_1^{IJ}A_2^{I\dot{A}} + A_3^{\dot{A}\dot{B}}A_2^{I\dot{B}}\right).$$

Varying $A_{1,2}$ in the potential, on the other hand, and making use of the above formulas (3.24) together with (3.20), we arrive at:

$$\overline{\chi}^{\dot{A}}\epsilon^K(\Gamma^K\Gamma^I)_{\dot{A}\dot{B}}\left(\frac{3}{16}\tilde{A}_1^{IJ}A_2^{J\dot{B}} - \frac{1}{16}\tilde{A}_3^{\dot{B}\dot{C}}A_2^{I\dot{C}}\right).$$

By the tracelessness of $A_2^{I\dot{A}}$ we can drop the tildes in this expression, and thus obtain the second relation

$$3A_1^{IJ}A_2^{J\dot{A}} - A_2^{I\dot{B}}A_3^{\dot{A}\dot{B}} = \frac{1}{16}\,(\Gamma^I\Gamma^J)_{\dot{A}\dot{B}}\left(3A_1^{JK}A_2^{K\dot{B}} - A_2^{J\dot{C}}A_3^{\dot{B}\dot{C}}\right), \tag{3.26}$$

which must be satisfied for local supersymmetry to hold.

Thus, at linear order in the fermions, supersymmetry requires the tensors $A_{1,2,3}$ to satisfy the identities (3.18), (3.19), and (3.22)–(3.26). However, these do not yet constitute a complete set of restrictions. In marked contrast to the $D \geq 4$ gauged supergravities, we get further and independent conditions at cubic order in the fermions. This special feature is again related to the algebraic independence of the third tensor A_3. Although the necessary calculations are quite tedious, we here refrain from giving details and simply state the results, as the relevant Fierz technology is (or should be) standard by now. Interested readers may find many relevant formulas in [22].

The analysis of the $(\overline{\psi}\psi)(\overline{\psi}\epsilon)$ terms gives

$$T_{IJ|KL} = 2\delta^{\overline{I[K}\,A_1^{L]J}} + T_{[IJ|KL]}\,. \tag{3.27}$$

The structure of the r.h.s. of this equation thus restricts $T_{IJ|KL}$ to the SO(16) components 1, 135 and 1820. Demanding the cancellation of $(\overline{\chi}\chi)(\overline{\psi}\epsilon)$ terms yields three more constraints:

$$A_3^{(0)} + 2A_1^{(0)} = 0\,, \qquad A^{(8)}_{3\,I_1\ldots I_8} = 0\,, \qquad T_{[IJ|KL]} = 2\tilde{A}^{(4)}_{3\,IJKL}\,, \tag{3.28}$$

such that with (3.19), (3.23), and (3.27) the T-tensor (3.21) may be entirely expressed in terms of the tensors $A_{1,2,3}$:

$$\begin{aligned} T_{IJ|KL} &= 2\,\delta^{IJ}_{KL}\,A_1^{(0)} + 2\,\overline{\delta^{\,I[K}\,\tilde{A}_1^{L]J}} + 2\,\tilde{A}^{(4)}_{3\,IJKL}\,, \\ T_{IJ|A} &= \Gamma^{[I}_{A\dot{A}}\,A_2^{J]\dot{A}}\,, \\ T_{A|B} &= -A_1^{(0)}\delta_{AB} + \frac{1}{2\cdot 4!}\,\Gamma^{IJKL}_{AB}\,\tilde{A}^{(4)}_{3\,IJKL}\,. \end{aligned} \tag{3.29}$$

In particular, the two singlets and the two 1820 representations in $T_{IJ|KL}$ and $T_{A|B}$ coincide. Finally, the analysis of the $(\overline{\chi}\chi)(\overline{\chi}\epsilon)$ terms yields

$$\delta A^{(4)}_{3\,IJKL} = -\frac{1}{2}\,\overline{\epsilon}^M\chi^{\dot{A}}\,\left(\Gamma^M\Gamma^{[IJK}\right)_{\dot{A}\dot{B}}\,A_2^{L]\dot{B}}\,. \tag{3.30}$$

In order to derive this condition and to prove the vanishing of the $(\overline{\chi}\chi)(\overline{\chi}\epsilon)$ terms, one needs the additional Fierz identity, which cannot be derived from the relations given in [22, appendix]

$$\begin{aligned} &(\overline{\chi}\Gamma^{KLMN}\chi)\,(\overline{\chi}^{\dot{A}}\epsilon^I)\,(\Gamma^I\Gamma^{KLM})_{\dot{A}\dot{B}}A_2^{N\dot{B}} = \\ &\quad = 36\,(\overline{\chi}\gamma_\mu\Gamma^{IJ}\chi)\,(\overline{\chi}^{\dot{A}}\gamma^\mu\epsilon^I)\,A_2^{J\dot{A}} - 4\,(\overline{\chi}\gamma_\mu\Gamma^{KL}\chi)\,(\overline{\chi}^{\dot{A}}\gamma^\mu\epsilon^I)\,\Gamma^{KL}_{\dot{A}\dot{B}}A_2^{I\dot{B}} + \\ &\qquad + 48\,(\overline{\chi}\chi)\,(\overline{\chi}^{\dot{A}}\epsilon^I)\,A_2^{I\dot{A}} - 12\,(\overline{\chi}\gamma_\mu\Gamma^{KL}\chi)\,(\overline{\chi}^{\dot{A}}\gamma^\mu\epsilon^I)\,\Gamma^{IK}_{\dot{A}\dot{B}}A_2^{L\dot{B}}\,. \end{aligned}$$

The tracelessness of $A_2^{I\dot{A}}$ is again crucial in obtaining this result.

Let us summarize our findings. The complete set of consistency conditions ensuring supersymmetry of the gauged lagrangian (3.9) is given by the linear relations (3.29), the differential identities (3.22), (3.30), the relation (3.18), and the quadratic identities (3.25), (3.26). The tensors $A_{1,2,3}$ can contain only the SO(16) representations $1, 135, 1820$ and $\overline{1920}$. Equations (3.29) show that likewise the T-tensor may contain only these representations. The remarkable fact — which eventually allows the resolution of all identities — is that these SO(16) representations combine into representations of $E_{8(8)}$. More specifically, we have

$$135 + 1820 + \overline{1920} = \mathbf{3875}\,, \tag{3.31}$$

while the first relation from (3.28) ensures that the two SO(16) singlets originate from one singlet of $E_{8(8)}$, such that the full $E_{8(8)}$ content of the tensors $A_{1,2,3}$ is contained in the $E_{8(8)}$ representations $\mathbf{1} + \mathbf{3875}$. Apart from the occurrence of an extra singlet, this fusion of tensors into representations of the hidden global $E_{d(d)}$ takes place already in dimensions $D = 4$ and $D = 5$, where the Yukawa couplings are given by tensors transforming in the **912** of $E_{7(7)}$ [33] and in the **351** of $E_{6(6)}$ [6], respectively. We shall come back to this point in the next section.

Perhaps the most unexpected feature of our construction is the fact that the vector fields appear via a CS term (3.10) in order g, rather than the standard Yang-Mills term. This has no analog in higher dimensions, where the vector fields appear already in the ungauged theory via an abelian kinetic term. In hindsight this coupling of the vector fields turns out to be the only consistent way to bring in the dual vector fields without introducing new propagating degrees of freedom, and thereby to preserve the balance of bosonic and fermionic physical degrees of freedom.

The emergence of non-abelian CS terms in the maximally supersymmetric theories naturally leads to a *non-abelian extension of the duality relation* (2.14)

$$\epsilon^{\mu\nu\rho}\widehat{\mathcal{B}}_{\mu\nu}{}^{m} = 2\, e\mathcal{V}^{m}{}_{A}\,\widehat{\mathcal{P}}^{\,\rho A} - \frac{\mathrm{i}}{2}\, e\mathcal{V}^{m}{}_{IJ}\,\overline{\chi}\gamma^{\rho}\Gamma^{IJ}\,\chi\,, \tag{3.32}$$

which consistently reduces to (2.14) in the limit $g \to 0$. However, in this limit, the vector fields drop from the lagrangian such that the duality relation (2.14) no longer follows from a variational principle in the ungauged theory but rather must be imposed by hand. This can be viewed as a very mild form of the gauge discontinuity encountered for gauged supergravities in odd dimensions [3, 4, 6]. In contrast to those models however, the lagrangian (3.9) has a perfectly smooth limit as $g \to 0$.

Because of the explicit appearance of the gauge fields on the r.h.s. of the non-abelian duality relation it is no longer possible to trade the vector fields for scalar fields and thereby eliminate them, unlike in [28]. Vice versa, the explicit appearance of the scalar fields in the potential of (3.9) also excludes the possibility to eliminate some of these fields by replacing them by vector fields. In contrast to the ungauged theory which allows for different equivalent formulations related by duality, the gauged theory apparently comes in a unique form which requires the maximal number of scalar fields together with the dual vectors corresponding to the gauge group G_0.

Note that unlike in (2.14), the nonabelian duality relation (3.32) may be imposed only for those vector fields which belong to the gauge group G_0. Having gauged the theory, we can no longer introduce additional vector fields as was the case for the ungauged theory. This is because additional vector fields transforming nontrivially under the gauge group G_0 would acquire mass terms in the gauged theory, entailing a mismatch between bosonic and fermionic degrees of freedom. As a consequence, (3.32) does not imply the full set of bosonic equations of motion, but just their projection onto the subgroup G_0. However, just as in (2.15) we may deduce the equations of motion for the vector fields from (3.32) by acting on both sides with

$\epsilon_{\rho\mu\nu}\mathcal{D}^\nu$ and making use of (3.7):

$$\begin{aligned}
\mathcal{D}_\nu \mathcal{B}^{\mu\nu m} &= \frac{1}{2}\, g e^{-1}\epsilon^{\mu\nu\rho} \left(\mathcal{V}^m{}_A \mathcal{V}^n{}_A + \mathcal{V}^m{}_{IJ}\mathcal{V}^n{}_{IJ}\right)\, \Theta_{nk}\, \mathcal{B}_{\nu\rho}{}^k - \\
&\quad - \frac{1}{2}\, e^{-1}\epsilon^{\mu\nu\rho}\, \mathcal{V}^m{}_{IJ}\, \mathcal{Q}^{IJ}_{\nu\rho} + \text{fermionic terms} \\
&= g\left(\mathcal{V}^m{}_B T_{B|A} + \mathcal{V}^m{}_{IJ} T_{IJ|A}\right) \mathcal{P}^{\mu A} - \frac{1}{2}\, e^{-1}\epsilon^{\mu\nu\rho}\, \mathcal{V}^m{}_{IJ}\, \mathcal{Q}^{IJ}_{\nu\rho} + \\
&\quad + \text{fermionic terms}\,.
\end{aligned} \tag{3.33}$$

4. T-identities

In the foregoing section we have derived the consistency conditions which must be satisfied by the tensors $A_{1,2,3}$ and the T-tensor in order to ensure the full supersymmetry of the gauged action (3.9). It remains to show that these conditions admit nontrivial solutions $A_{1,2,3}(\mathcal{V})$. This will single out the possible gauge groups $G_0 \subset E_{8(8)}$. Recall that in the three dimensional model the choice of gauge group is less restricted than in higher dimensions where the gauge group $G_0 \subset G$ is essentially determined by the fact that a maximal subset of the vector fields of the theory must transform in its adjoint representation.

Up to this point, we have made no assumptions on the gauge group $G_0 \subset E_{8(8)}$, which is characterized by its embedding tensor $\Theta_{\mathcal{MN}}$, cf. (3.2). We will now show that all the consistency conditions derived in the previous section may be encoded into a single algebraic equation for the embedding tensor.

According to (3.3), $\Theta_{\mathcal{MN}}$ transforms in the symmetric tensor product

$$(\mathbf{248}\times\mathbf{248})_{\rm sym} = \mathbf{1} + \mathbf{3875} + \mathbf{27000}\,. \tag{4.1}$$

The explicit projectors of this decomposition have been computed in [34]

$$\begin{aligned}
(\mathbb{P}_{\mathbf{1}})_{\mathcal{MN}}{}^{\mathcal{KL}} &= \frac{1}{248}\,\eta_{\mathcal{MN}}\,\eta^{\mathcal{KL}}\,, \\
(\mathbb{P}_{\mathbf{3875}})_{\mathcal{MN}}{}^{\mathcal{KL}} &= \frac{1}{7}\,\delta^{\mathcal{K}}_{(\mathcal{M}}\delta^{\mathcal{L}}_{\mathcal{N})} - \frac{1}{56}\,\eta_{\mathcal{MN}}\,\eta^{\mathcal{KL}} - \frac{1}{14}\, f^{\mathcal{P}}{}_{\mathcal{M}}{}^{(\mathcal{K}} f_{\mathcal{PN}}{}^{\mathcal{L})}\,, \\
(\mathbb{P}_{\mathbf{27000}})_{\mathcal{MN}}{}^{\mathcal{KL}} &= \frac{6}{7}\,\delta^{\mathcal{K}}_{(\mathcal{M}}\delta^{\mathcal{L}}_{\mathcal{N})} + \frac{3}{217}\,\eta_{\mathcal{MN}}\,\eta^{\mathcal{KL}} + \frac{1}{14}\, f^{\mathcal{P}}{}_{\mathcal{M}}{}^{(\mathcal{K}} f_{\mathcal{PN}}{}^{\mathcal{L})}\,.
\end{aligned} \tag{4.2}$$

Accordingly, $\Theta_{\mathcal{MN}}$ may be decomposed as

$$\Theta_{\mathcal{MN}} = \theta\,\eta_{\mathcal{MN}} + \Theta^{\mathbf{3875}}_{\mathcal{MN}} + \Theta^{\mathbf{27000}}_{\mathcal{MN}}\,, \tag{4.3}$$

with

$$\Theta^{\mathbf{3875}}_{\mathcal{MN}} = (\mathbb{P}_{\mathbf{3875}})_{\mathcal{MN}}{}^{\mathcal{KL}}\,\Theta_{\mathcal{KL}}\,, \quad \Theta^{\mathbf{27000}}_{\mathcal{MN}} = (\mathbb{P}_{\mathbf{27000}})_{\mathcal{MN}}{}^{\mathcal{KL}}\,\Theta_{\mathcal{KL}}\,.$$

The T-tensor as it has been defined in (3.21) is given by a rotation of $\Theta_{\mathcal{MN}}$ by the matrix $\mathcal{V}$. It may likewise be decomposed

$$T_{\mathcal{A}|\mathcal{B}} = T^{\mathbf{1}}_{\mathcal{A}|\mathcal{B}} + T^{\mathbf{3875}}_{\mathcal{A}|\mathcal{B}} + T^{\mathbf{27000}}_{\mathcal{A}|\mathcal{B}}\,, \tag{4.4}$$

with

$$T^{\mathbf{3875}}_{\mathcal{A}|\mathcal{B}} = (\mathbb{P}_{\mathbf{3875}})_{\mathcal{AB}}{}^{\mathcal{CD}}\, T_{\mathcal{C}|\mathcal{D}} = \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}} \mathcal{V}^{\mathcal{N}}{}_{\mathcal{B}}\, \Theta^{\mathbf{3875}}_{\mathcal{MN}}\,, \qquad \text{etc.}$$

where the second equality is due to invariance of the projectors under $E_{8(8)}$. Analogous tensors have been defined in [2] and [6] for the maximally gauged models in $D = 4$ and $D = 5$, respectively. Unlike those T-tensors, however, the T-tensor here is quadratic in $\mathcal{V}$, as already emphasized before.

4.1 The constraint for the embedding tensor

We have seen that supersymmetry of the gauged lagrangian in particular implies the set of relations (3.29) for the T-tensor. As discussed above, these relations show that T may only contain the SO(16) representations contained in the $\mathbf{1} + \mathbf{3875}$ of $E_{8(8)}$. It follows that equations (3.29) can be solved for $A_{1,2,3}$ if and only if

$$T^{\mathbf{27000}}_{\mathcal{A}|\mathcal{B}} = 0 \iff \Theta^{\mathbf{27000}}_{\mathcal{MN}} = 0\,. \tag{4.5}$$

This is a set of linear algebraic equations for the embedding tensor $\Theta_{\mathcal{MN}}$. We stress once more the remarkable fact that the equations (3.29) combine into an $E_{8(8)}$ covariant condition for the T-tensor which makes it possible to translate these equations into a condition for the constant tensor $\Theta_{\mathcal{MN}}$. In particular, each single equation from (3.29) yields an SO(16) covariant restriction on the T-tensor (3.21) which already implies the full set of relations (3.29), if it is to be satisfied for all $E_{8(8)}$ valued matrices $\mathcal{V}$.

We shall show in the following sections that (4.5) not only reproduces the linear equations (3.29) but indeed implies the complete set of consistency conditions (including the differential and quadratic ones) identified in the last section.[4]

4.2 Linear identities

Making use of the explicit form of the projectors (4.2), equation (4.5) takes the form

$$\begin{aligned}
\Theta_{IJ,KL} &= -\frac{2}{7}\,\delta_{\underbracket{I[K}\,\Theta_{L]M,MJ}} + \Theta_{[IJ,KL]} + \frac{16}{7}\,\theta\,\delta^{IJ}_{KL}\,, \\
\Theta_{IJ,A} &= \frac{1}{7}\,(\Gamma_{\underbracket{I}\Gamma^{L})_{AB}\,\Theta_{B,LJ}}\,, \\
\Theta_{A,B} &= \frac{1}{96}\,\Gamma^{IJKL}_{AB}\,\Theta_{IJ,KL} + \theta\,\delta_{AB}\,,
\end{aligned} \tag{4.6}$$

and likewise for T. These equations contain the complete set of linear identities among different components of the T-tensor. Once they are satisfied, the T-tensor

[4] Let us stress once more that in addition to (4.5), Θ must project onto a subgroup. If that condition is dropped, further solutions to (4.5) can be found, but the T-tensor would then fail to satisfy the quadratic identities of section 4.4.

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may be entirely expressed in terms of the tensors $A_{1,2,3}$ as found in (3.29) above:

$$\begin{aligned}
T_{IJ|KL} &= 2\,\delta^{\,I[K}\,A_1^{L]J} + \frac{1}{64}\,\Gamma^{IJKL}_{\dot A\dot B}\,A_3^{\dot A\dot B}\,,\\
T_{IJ|A} &= \Gamma^{[I}_{A\dot A}\,A_2^{J]\dot A}\,,\\
T_{A|B} &= \frac{1}{6144}\,\Gamma^{IJKL}_{AB}\,\Gamma^{IJKL}_{\dot A\dot B}\,A_3^{\dot A\dot B} + \theta\,\delta_{AB}\,.
\end{aligned} \tag{4.7}$$

These equations may be inverted and give the solution for the tensors $A_{1,2,3}$ in terms of the T-tensor:

$$\begin{aligned}
A_1^{IJ} &= \frac{8}{7}\,\theta\,\delta_{IJ} + \frac{1}{7}\,T_{IK|JK}\,,\\
A_2^{I\dot A} &= -\frac{1}{7}\,\Gamma^{J}_{A\dot A}\,T_{IJ|A}\,,\\
A_3^{\dot A\dot B} &= 2\theta\,\delta_{\dot A\dot B} + \frac{1}{48}\,\Gamma^{IJKL}_{\dot A\dot B}\,T_{IJ|KL}\,.
\end{aligned} \tag{4.8}$$

4.3 Differential identities

With the linear identities derived in the last section we may now compute the variation of the tensors $A_{1,2,3}$ when $\mathcal{V}$ is varied. Since the matrix $\mathcal{V}$ lives in the adjoint representation, its variation along an invariant vector field Σ^A is given by

$$\frac{\delta \mathcal{V}^{\mathcal{M}}{}_{\mathcal{B}}}{\delta\Sigma^A} = f_{\mathcal{B}}{}^{\mathcal{C}A}\,\mathcal{V}^{\mathcal{M}}{}_{\mathcal{C}} \;\Longrightarrow\; \left\{ \begin{aligned} \frac{\delta\mathcal{V}^{\mathcal{M}}{}_{IJ}}{\delta\Sigma^A} &= -\frac{1}{2}\,\Gamma^{IJ}_{AB}\,\mathcal{V}^{\mathcal{M}}{}_{B}\\ \frac{\delta\mathcal{V}^{\mathcal{M}}{}_{B}}{\delta\Sigma^A} &= -\frac{1}{4}\,\Gamma^{IJ}_{AB}\,\mathcal{V}^{\mathcal{M}}{}_{IJ} \end{aligned}\right. . \tag{4.9}$$

From (4.8) we then obtain

$$\begin{aligned}
\frac{\delta A_1^{IJ}}{\delta\Sigma^A} &= \frac{1}{14}\,\left(\Gamma^{IK}_{AB}\,T_{KJ|B} + \Gamma^{JK}_{AB}\,T_{KI|B}\right),\\
\frac{\delta A_2^{I\dot A}}{\delta\Sigma^A} &= \frac{1}{14}\,\Gamma^{J}_{B\dot A}\,\left(\Gamma^{IJ}_{AC}\,T_{B|C} + \frac{1}{2}\,\Gamma^{MN}_{AB}\,T_{IJ|MN}\right),\\
\frac{\delta A_3^{\dot A\dot B}}{\delta\Sigma^A} &= -\frac{1}{48}\,\Gamma^{IJKL}_{\dot A\dot B}\,\Gamma^{KL}_{AB}\,T_{IJ|B}\,.
\end{aligned}$$

Rewriting the expressions on the r.h.s. in terms of the tensors $A_{1,2,3}$ by means of (4.7) we get

$$\begin{aligned}
\frac{\delta A_1^{IJ}}{\delta\Sigma^A} &= \Gamma^{(I}_{A\dot A}\,A_2^{J)\dot A}\,,\\
\frac{\delta A_2^{I\dot A}}{\delta\Sigma^A} &= \frac{1}{2}\,\left(\Gamma^{M}_{A\dot A}\,A_1^{IM} + \Gamma^{I}_{A\dot B}\,A_3^{\dot A\dot B} - \Gamma^{I}_{B\dot A}\,T_{A|B}\right),\\
\frac{\delta A_3^{\dot A\dot B}}{\delta\Sigma^A} &= \frac{1}{48}\,\Gamma^{IKMN}_{\dot A\dot B}\,\Gamma^{KMN}_{A\dot C}\,A_2^{I\dot C}\,.
\end{aligned} \tag{4.10}$$

This reproduces equations (3.24) and (3.30) from the last section. In particular, we obtain the covariant derivatives of the tensors $A_{1,2}$

$$\begin{aligned}
\mathcal{D}_\mu A_1^{IJ} &= \Gamma^{(I}_{A\dot{A}}\, A_2^{J)\dot{A}}\, \mathcal{P}_\mu^A\,, \\
\mathcal{D}_\mu A_2^{I\dot{A}} &= \frac{1}{2}\left(\Gamma^M_{A\dot{A}}\, A_1^{IM} + \Gamma^I_{A\dot{B}}\, A_3^{\dot{A}\dot{B}} - \Gamma^I_{B\dot{A}}\, T_{A|B}\right) \mathcal{P}_\mu^A\,,
\end{aligned} \tag{4.11}$$

which coincide with equations (3.22) found before. The variation (4.10) further allows to compute the variation of the scalar potential (3.12)

$$\frac{\delta}{\delta\Sigma^A}\left(A_1^{IJ}A_1^{IJ} - \frac{1}{2}\, A_2^{I\dot{A}}A_2^{I\dot{A}}\right) = \frac{1}{2}\, \Gamma^M_{A\dot{A}}\left(3A_1^{MN}A_2^{N\dot{A}} - A_3^{\dot{A}\dot{B}}A_2^{M\dot{B}}\right),$$

which has also been used in the last section. Together with the quadratic identity (4.20) to be derived below, this yields the condition for stationary points of the potential

$$\frac{\delta W}{\delta\Sigma^A} = 0 \iff 3\, A_1^{IM}A_2^{M\dot{A}} = A_3^{\dot{A}\dot{B}}A_2^{I\dot{B}}\,. \tag{4.12}$$

Obviously, a sufficient condition for stationarity is $A_2^{I\dot{A}} = 0$.

4.4 Quadratic identities

So far, we have exploited the projector condition (4.5) to derive linear identities in $T_{\mathcal{A}|\mathcal{B}}$. However, additional information stems from the fact that the tensor $\Theta_{\mathcal{M}\mathcal{N}}$ is built from projectors onto subgroups, cf. (3.2). This can be used to derive further identities quadratic in the tensors $A_{1,2,3}$. As we have seen in the previous section, identities of this type are also needed to ensure supersymmetry of the gauged theory.

Since $\Theta_{\mathcal{M}\mathcal{N}}$ projects onto a subgroup $G_0 \subset G$, it satisfies:

$$\Theta_{\mathcal{K}(\mathcal{M}}\, f_{\mathcal{N})}{}^{\mathcal{K}\mathcal{L}}\, \Theta_{\mathcal{L}\mathcal{P}} = 0\,, \tag{4.13}$$

which follow from closure of G_0 and the antisymmetry of the structure constants. Invariance of the structure constants then implies

$$\Theta_{mn}\, \mathcal{V}^n{}_{\mathcal{C}}\, f^{\mathcal{C}\mathcal{D}}{}_{(\mathcal{A}}\, T_{\mathcal{B})|\mathcal{D}} = 0\,. \tag{4.14}$$

Evaluate this expression for $(\mathcal{A},\mathcal{B}) = ([IM],[KM])$:

$$4\, \mathcal{V}^m{}_{N(I}\, T_{K)M|MN} + \Gamma^{IM}_{AB}\, \mathcal{V}^m{}_A\, T_{KM|B} + \Gamma^{KM}_{AB}\, \mathcal{V}^m{}_A\, T_{IM|B} = 0\,,$$

where the index m is projected onto the subalgebra $\mathfrak{g}_0$. Inserting (4.7) yields

$$\mathcal{V}^m{}_A\, \Gamma^{(I}_{A\dot{A}}\, A_2^{K)\dot{A}} = \mathcal{V}^m{}_{IM}\, A_1^{MK} + \mathcal{V}^m{}_{KM}\, A_1^{MI}\,, \tag{4.15}$$

and thus the identity (3.18), required above for closure of the supersymmetry algebra in the gauged theory. If we contract this equation with $\mathcal{V}^n{}_{JK}\Theta_{mn}$, symmetrize in (IJ) and once more insert (4.7), we obtain

$$A_1^{IK}A_1^{KJ} - \frac{1}{2}\, A_2^{I\dot{A}}A_2^{J\dot{A}} = \frac{1}{16}\, \delta^{IJ}\left(A_1^{KL}A_1^{KL} - \frac{1}{2}\, A_2^{K\dot{A}}A_2^{K\dot{A}}\right). \tag{4.16}$$

This gives already the quadratic identity (3.25). If on the other hand we contract (4.15) with $\Gamma^K_{B\dot{A}}\mathcal{V}^n{}_B\Theta_{mn}$, we obtain after inserting (4.7)

$$\begin{aligned}\frac{1}{64}\,\Gamma^{IKMN}_{\dot{C}\dot{D}}\,\Gamma^{MN}_{\dot{A}\dot{B}}\,A_2^{K\dot{B}}A_3^{\dot{C}\dot{D}} &= -32\,A_1^{IN}A_2^{N\dot{A}} + 2\,(\Gamma^I\Gamma^K)_{\dot{A}\dot{B}}\,A_1^{KN}A_2^{N\dot{B}} - 16\,\theta\,A_2^{I\dot{A}} + \\ &\quad + 10\,A_2^{I\dot{B}}A_3^{\dot{A}\dot{B}} - (\Gamma^I\Gamma^K)_{\dot{A}\dot{B}}\,A_2^{K\dot{C}}A_3^{\dot{B}\dot{C}}. \qquad (4.17)\end{aligned}$$

Evaluating (4.14) for $(\mathcal{A},\mathcal{B}) = ([IJ],A)$ and contracting with $\Gamma^J_{A\dot{A}}$ leads to

$$\begin{aligned}\frac{1}{6}\,(\Gamma^J\Gamma^{MNKL})_{A\dot{A}}\,\mathcal{V}^m{}_A\,T_{MN|KL} - \frac{1}{12}\,(\Gamma^{MNKL}\Gamma^J)_{A\dot{A}}\,\mathcal{V}^m{}_A\,T_{MN|KL} = \\ = \frac{4}{7}\,(\Gamma^K\Gamma^{MN})_{A\dot{A}}\,\mathcal{V}^m{}_{MN}\,T_{JK|A} - \frac{16}{7}\,\Gamma^K_{A\dot{A}}\,\mathcal{V}^m{}_{JM}\,T_{MK|A} + \\ + \frac{8}{7}\,\Gamma^K_{A\dot{A}}\,\mathcal{V}^m{}_A\,T_{JM|MK} + \frac{1}{14}\,\Gamma^J_{A\dot{A}}\,\mathcal{V}^m{}_A\,T_{MN|MN}\,, \qquad (4.18)\end{aligned}$$

again, if the index m is projected onto the subalgebra $\mathfrak{g}_0$. To obtain the desired identity, we contract this equation with $\mathcal{V}^n{}_{IJ}\Theta_{mn}$ and insert (4.7). After some calculation we arrive at

$$\begin{aligned}\frac{1}{64}\,\Gamma^{IKMN}_{\dot{C}\dot{D}}\,\Gamma^{MN}_{\dot{A}\dot{B}}\,A_2^{K\dot{B}}A_3^{\dot{C}\dot{D}} &= 64\,A_1^{IN}A_2^{N\dot{A}} - 4\,(\Gamma^I\Gamma^K)_{\dot{A}\dot{B}}\,A_1^{KN}A_2^{N\dot{B}} - 16\,\theta\,A_2^{I\dot{A}} - \\ &\quad - 22\,A_2^{I\dot{B}}A_3^{\dot{A}\dot{B}} + (\Gamma^I\Gamma^K)_{\dot{A}\dot{B}}\,A_2^{K\dot{C}}A_3^{\dot{B}\dot{C}}\,. \qquad (4.19)\end{aligned}$$

Equating (4.17) and (4.19), we finally obtain

$$3A_1^{IJ}A_2^{J\dot{A}} - A_2^{I\dot{B}}A_3^{\dot{A}\dot{B}} = \frac{1}{16}\,(\Gamma^I\Gamma^J)_{\dot{A}\dot{B}}\left(3A_1^{JK}A_2^{K\dot{B}} - A_2^{J\dot{C}}A_3^{\dot{B}\dot{C}}\right). \qquad (4.20)$$

We have thus shown that the condition (4.5) together with the fact that $\Theta_{\mathcal{MN}}$ projects onto a subalgebra implies the quadratic identities (4.16) and (4.20) which coincide with (3.25), (3.26) found above. Altogether, we recover in this fashion all the identities required in section 3 from the single projector condition (4.5) for the embedding tensor $\Theta_{\mathcal{MN}}$.

5. Admissible gauge groups G_0

Having reduced the consistency conditions required by local supersymmetry to a set of algebraic conditions (4.5) for the embedding tensor of the gauge group $G_0 \subset G$, we must now ascertain that this condition admits non-trivial solutions and classify them. This is the objective of the present section. As we will see the variety of solutions of (4.5), each of which gives rise to a maximally supersymmetric gauged supergravity, is far richer than in dimensions $D \geq 4$.

The power of equation (4.5) is based on its formulation as a single projector condition in the tensor product decomposition (4.1). This permits the construction of solutions by purely group theoretical means. To demonstrate that these methods also clarify the structure of the T-identities in $D \geq 4$, we derive the analog of (4.5) to re-obtain the results of [2] and [6]. Group theoretical arguments then show immediately that the gauge groups SO(8) and SO(6), respectively, solve the relevant equations. In particular, this provides a unifying argument for the consistency of all the noncompact gaugings found subsequently in [35, 36, 6].

The analysis for three dimensions turns out to be more involved, but extending the above arguments we arrive at a variety of admissible gauge groups. There is a regular series of gauge groups $\mathrm{SO}(p, 8-p) \times \mathrm{SO}(p, 8-p)$ including the maximal compact $\mathrm{SO}(8) \times \mathrm{SO}(8)$, and several exceptional noncompact gauge groups, summarized in table 2 below. Still this is not a complete classification of admissible gauge groups, as we restrict the analysis of compact and noncompact gauge groups to the maximal subgroups of SO(16) and $E_{8(8)}$, respectively. We leave the exploration of smaller rank gauge groups to future work.

5.1 T-identities and gauge groups in higher dimensions

As a "warm-up" let us first apply our techniques to the gauged maximal supergravities in $D = 4, 5$. This will allow us to shortcut the derivation of the (linear) T-identities given in the original work.

5.1.1 $D = 4$

Like (4.4), the $D = 4$ T-tensor is obtained from a constant G_0-invariant tensor Θ by a field dependent rotation with the matrix $\mathcal{V} \in E_{7(7)}$ in the fundamental representation. The constant tensor Θ there transforms in the product of the adjoint and the fundamental representation

$$\mathbf{56} \times \mathbf{133} = \mathbf{56} + \mathbf{912} + \mathbf{6480}\,, \tag{5.1}$$

of $E_{7(7)}$,[5] such that T is cubic rather than quadratic in the matrix entries of $\mathcal{V}$. Computations similar to those presented in the last section then show that full supersymmetry of the gauged lagrangian is equivalent to

$$T = T^{\mathbf{912}} \iff \Theta = \Theta^{\mathbf{912}}\,, \tag{5.2}$$

providing the analogue of (4.5). It is now straightforward to see that $G_0 = \mathrm{SO}(8)$ indeed gives a solution to (5.2): consider the decomposition of (5.1) under SO(8):[6]

$$\begin{aligned} \mathbf{56} &\rightarrow 2 \cdot 28\,, \\ \mathbf{912} &\rightarrow 2 \cdot 1 + 2 \cdot 35_v + 2 \cdot 35_s + 2 \cdot 35_c + \cdots\,, \\ \mathbf{6480} &\rightarrow 6 \cdot 28 + 2 \cdot 35_v + 2 \cdot 35_s + 2 \cdot 35_c + \cdots\,. \end{aligned} \tag{5.3}$$

[5] It is only for $E_{8(8)}$ that the fundamental representation coincides with the adjoint representation and the tensor Θ hence coincides with the embedding tensor of the group G_0.

[6] LiE [37] has been very helpful to quickly determine these decompositions.

As the singlets appear only in the **912**, any SO(8) invariant tensor in (5.1) automatically satisfies (5.2). The same argument proves the consistency of the noncompact SO$(p, 8-p)$ gaugings found in [36]. As shown in [38] equation (5.2) indeed contains no other solutions than those found in [2, 36].

5.1.2 $D = 5$

For $D = 5$, the constant tensor Θ transforms in the product of the adjoint and the fundamental representation

$$\mathbf{27} \times \mathbf{78} = \mathbf{27} + \mathbf{351} + \mathbf{1728}\,, \tag{5.4}$$

of $E_{6(6)}$. Rotation by $\mathcal{V}$ in the fundamental representation of $E_{6(6)}$ converts Θ into the T-tensor, cubic in the matrix entries of $\mathcal{V}$. Supersymmetry of the gauged lagrangian then is shown to be equivalent to

$$T = T^{\mathbf{351}} \quad \Longleftrightarrow \quad \Theta = \Theta^{\mathbf{351}}\,, \tag{5.5}$$

in analogy with (4.5) and (5.2). Again, it is straightforward to see that $G_0 = \mathrm{SO}(6)$ yields a solution to (5.5): under SO(6), (5.4) decomposes as

$$\begin{aligned} \mathbf{27} &\to 2\cdot 6 + 15\,, \\ \mathbf{351} &\to 1 + 2\cdot 6 + 2\cdot 10 + 2\cdot \overline{10} + 4\cdot 15 + \cdots\,, \\ \mathbf{1728} &\to 10\cdot 6 + 2\cdot 10 + 2\cdot \overline{10} + 9\cdot 15 + \cdots\,. \end{aligned} \tag{5.6}$$

Now the singlet appears only in the **351**, hence there is just one SO(6) invariant tensor in (5.4) which automatically satisfies (5.5). As before, this argument generalizes to all the noncompact gauge groups found in [6].

5.2 Compact gauge groups

Let us now come back to (4.5). We will first consider compact gauge groups $G_0 \subset \mathrm{SO}(16)$. Their embedding tensors satisfy

$$\Theta_{IJ,A} = 0 = \Theta_{A,B}\;; \tag{5.7}$$

the only nonvanishing component is $\Theta_{IJ,KL}$ which under SO(16) decomposes as

$$\Theta_{IJ,KL} \;\sim\; 1 + 135 + 1820 + 5304\,. \tag{5.8}$$

According to (3.31), the 5304 is part of the **27000** and must vanish for (4.5) to be satisfied. From (4.6) it further follows that the 1 and the 1820 coincide with the corresponding parts in $\Theta_{A,B}$ and thus must vanish due to (5.7). Hence, for compact G_0, only the 135 representation survives, and the condition (4.5) reduces to

$$\Theta_{IJ,KL} = \delta_{\underline{I[K}\,\Xi_{L]\underline{J}}}\,, \qquad \text{with} \quad \Xi_{IJ} = \frac{7}{2}\,\Theta_{IK,JK}\,, \quad \Xi_{II} = 0\,. \tag{5.9}$$

The tracelessness of Θ in particular rules out any simple compact gauge group.

In principle, the elementary form of the constraint (5.9) should allow a complete classification of the possible compact gauge groups; however, in the following, we restrict attention to the maximal subgroups of SO(16). They are

$$\begin{aligned}&\mathrm{SO}(9)\,, \quad \mathrm{SO}(5)\times\mathrm{SO}(5)\,, \quad \mathrm{SO}(3)\times\mathrm{USp}(8)\,,\\ &\text{and} \quad \mathrm{SO}(p)\times\mathrm{SO}(16-p)\,, \quad \text{for} \quad p=0,\ldots,8\,.\end{aligned} \tag{5.10}$$

A necessary condition for a compact gauge group to be admissible immediately follows from (5.9): there must exist a G_0-invariant tensor Ξ_{IJ} in the 135 of SO(16). In other words, there must be a singlet in the decomposition of 135 w.r.t. G_0. From the maximal subgroups (5.10) this already rules out the first three. It remains to study the $\mathrm{SO}(p)\times\mathrm{SO}(16-p)$. These groups have a unique invariant tensor in the 135:

$$\Xi_{ij} = (16-p)\,\delta_{ij}\,, \qquad \Xi_{\bar{i}\bar{j}} = -p\,\delta_{\bar{i}\bar{j}}\,, \tag{5.11}$$

where $i,j=1,\ldots p$ and $\bar{i},\bar{j}=p+1,\ldots,16$ denote the splitting of the SO(16) vector indices I, and the relative factor between Ξ_{ij} and $\Xi_{\bar{i}\bar{j}}$ is determined from tracelessness. By (5.9), the tensor $\Theta_{IJ,KL}$ satisfying (4.5) is

$$\Theta_{ij,kl} = (16-p)\,\delta^{ij}_{kl}\,, \quad \Theta_{\bar{i}\bar{j},\bar{k}\bar{l}} = -p\,\delta^{\bar{i}\bar{j}}_{\bar{k}\bar{l}}\,, \quad \Theta_{i\bar{j},k\bar{l}} = \frac{1}{2}(8-p)\,\delta_{ik}\,\delta_{\bar{j}\bar{l}}\,.$$

However, due to the nonvanishing mixed components $\Theta_{i\bar{j},k\bar{l}}$, this tensor coincides with the embedding tensor of $\mathrm{SO}(p)\times\mathrm{SO}(16-p)$ if and only if $p=8$. Hence we have shown that the only maximal subgroup of SO(16) whose embedding tensor satisfies the condition (4.5) is

$$G_0 = \mathrm{SO}(8)\times\mathrm{SO}(8) \subset \mathrm{SO}(16)\,, \tag{5.12}$$

where the ratio of coupling constants of the two factors is $g_1/g_2=-1$; in particular the trace part θ of $\Theta_{\mathcal{MN}}$ vanishes. Combining this with the results of the previous sections, we have thus shown the existence of a maximally supersymmetric gauged supergravity with compact gauge group $G_0=\mathrm{SO}(8)\times\mathrm{SO}(8)$. Under G_0, the scalar degrees of freedom decompose as

$$120 \to (1,28)+(28,1)+(8_s,8_c)\,, \quad 128 \to (8_v,8_v)+(8_c,8_s)\,, \tag{5.13}$$

while the spinors split into

$$16 \to (1,8_c)+(8_s,1)\,, \qquad \overline{128} \to (8_v,8_s)+(8_c,8_v)\,. \tag{5.14}$$

Amongst other things we here recognize the standard decomposition of the on-shell IIA supergravity multiplets in terms of left and right moving string states.

5.3 Regular noncompact gauge groups

In order to identify the allowed noncompact gauge groups, we first recall that for the maximal gauged supergravity in $D = 4$, several noncompact gaugings were found by analytic continuation [35, 36]. The noncompact gauge groups are thus alternative real forms of the complexified gauge group $\mathrm{SO}(8, \mathbb{C})$, and the consistency of the noncompact gaugings was basically a consequence of the consistency of the original theory [2] with compact gauge group. The results of the last section suggest that analogous gaugings should exist for the different real forms of (5.12).

The complexification of (5.12) is $\mathrm{SO}(8, \mathbb{C}) \times \mathrm{SO}(8, \mathbb{C})$. Its real forms which are also contained in $E_{8(8)}$ are given by

$$G_0 = \mathrm{SO}(p, 8-p)^{(1)} \times \mathrm{SO}(p, 8-p)^{(2)}\,, \qquad \text{for } p = 1, \ldots, 4\,. \tag{5.15}$$

They are embedded in $E_{8(8)}$ via the maximal noncompact subgroup $\mathrm{SO}(8, 8)$. Therefore the latter group is the analogue of the subgroups $\mathrm{SL}(8, \mathbb{R}) \subset E_{7(7)}$ in $D = 4$ and $\mathrm{SL}(6, \mathbb{R}) \times \mathrm{SL}(2, \mathbb{R}) \subset E_{6(6)}$ in $D = 5$. To further illustrate the embedding, we have denoted the two factors of G_0 by superscripts $_{(1)}$, $_{(2)}$ whereas we denote the two factors of (5.12) by subscripts $_{L,R}$. The maximal compact subgroup of (5.15) is given by

$$\begin{aligned} H_0 &= H^{(1)} \times H^{(2)} \\ &\equiv \Big(\mathrm{SO}(p)^{(1)}_L \times \mathrm{SO}(8-p)^{(1)}_R\Big) \times \Big(\mathrm{SO}(p)^{(2)}_R \times \mathrm{SO}(8-p)^{(2)}_L\Big)\,, \end{aligned} \tag{5.16}$$

with

$$\begin{aligned} &H^{(1)} \subset \mathrm{SO}(p, 8-p)^{(1)}\,, \;\; \mathrm{SO}(p)^{(1)}_L \times \mathrm{SO}(8-p)^{(2)}_L \subset \mathrm{SO}(8)_L\,, \\ &H^{(2)} \subset \mathrm{SO}(p, 8-p)^{(2)}\,, \;\; \mathrm{SO}(p)^{(1)}_R \times \mathrm{SO}(8-p)^{(2)}_R \subset \mathrm{SO}(8)_R\,. \end{aligned}$$

The embedding of H_0 into $\mathrm{SO}(8)_L \times \mathrm{SO}(8)_R$ is the standard one, without any triality rotation. In other words, the 8_v of $\mathrm{SO}(8)_L$ decomposes into $(p, 1) + (1, 8-p)$ under $\mathrm{SO}(p)^{(1)}_L \times \mathrm{SO}(8-p)^{(2)}_L$, etc.

Consistency of the gauged theories with noncompact gauge groups (5.15) could in principle be shown in analogy with [36, 39] by the method of analytic continuation. Alternatively, their consistency follows from an algebraic argument along the lines of the last section by use of our form of the consistency condition (4.5). This gives the analogue of the noncompact gaugings found in higher dimensions [36, 6].

5.4 Exceptional noncompact gauge groups

Next, we discuss noncompact gauge groups which unlike the groups identified in (5.15) do not share the same complexification with any compact subgroup contained in $E_{8(8)}$. Their existence is again a consequence of the absence of any a priori restriction on the number of vector fields in three dimensions.

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These noncompact solutions to (4.5) may be found by a purely group theoretical argument. As an example, consider the maximal subgroup $G_0 = G_{2(2)} \times F_{4(4)}$. Under G_0 the adjoint representation of $E_{8(8)}$ decomposes as

$$\mathbf{248} \to (14, 1) + (1, 52) + (7, 26)\,. \tag{5.17}$$

Accordingly, the symmetric tensor product (4.1) contains three singlets under G_0, and the Cartan-Killing form of $E_{8(8)}$ decomposes into three G_0-invariant tensors:

$$\eta_{\mathcal{MN}} = \eta^{(14,1)}_{\mathcal{MN}} + \eta^{(1,52)}_{\mathcal{MN}} + \eta^{(7,26)}_{\mathcal{MN}}\,. \tag{5.18}$$

More precisely, each of the three terms on the r.h.s. of (4.1) contains exactly one singlet under G_0 [37]. Consequently, there is a linear combination

$$\alpha_1\, \eta^{(14,1)}_{\mathcal{MN}} + \alpha_2\, \eta^{(1,52)}_{\mathcal{MN}} + \alpha_3\, \eta^{(7,26)}_{\mathcal{MN}}\,,$$

which lies entirely in the **3875**. Subtracting a proper multiple of the $E_{8(8)}$ singlet (5.18), we find that

$$\Theta_{\mathcal{MN}} \equiv (\alpha_1 - \alpha_3)\, \eta^{(14,1)}_{\mathcal{MN}} + (\alpha_2 - \alpha_3)\, \eta^{(1,52)}_{\mathcal{MN}}\,, \tag{5.19}$$

satisfies (4.5). This is the embedding tensor of $G_0 = G_{2(2)} \times F_{4(4)}$ with a fixed ratio of coupling constants between the two factors, which solves (4.5) and (4.13). The results of the last section then prove the existence of a maximally supersymmetric gauged theory with gauge group $G_{2(2)} \times F_{4(4)}$.

The same argument may be applied to other noncompact subgroups of $E_{8(8)}$. A closer inspection of the above proof reveals that only two ingredients were needed, namely (i) that the gauge group G_0 consists of two simple factors and (ii) that the $E_{8(8)}$ representations **3875** and **27000** each contain precisely one singlet in the decomposition under G_0. As it turns out, this requirement is also met by the noncompact groups $E_{7(7)} \times \mathrm{SL}(2)$, $E_{6(6)} \times \mathrm{SL}(3)$, and all their real forms which are contained in $E_{8(8)}$. The list of exceptional noncompact subgroups passing this test, together with their maximal compact subgroups is displayed in table 2.

There are also real forms of these exceptional gauge groups — the compact forms of E_d for $d = 6, 7, 8$, and the real forms $E_{8(-24)}, E_{7(-25)}$ and $E_{6(-26)}$ — which are not contained in $E_{8(8)}$ and thus do not appear in this list. However, every real form that may be embedded in $E_{8(8)}$ gives rise to a maximally supersymmetric gauged supergravity. The "extremal" noncompact solution to (4.5) is given by the group $G_0 = E_{8(8)}$ itself, in which case $\Theta_{\mathcal{MN}}$ reduces to the Cartan-Killing form $\eta_{\mathcal{MN}}$.

To complete the construction of the theories with gauge groups given in table 2, it remains to compute the ratio of coupling constants between the two factors of G_0 which came out to be fixed to a specific value in (5.19). To this end, let us consider the general situation of a gauge group with two simple factors $G_0 = G^{(1)} \times G^{(2)}$, such

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$G_0 = G^{(1)} \times G^{(2)}$	maximal compact subgroup $H_0 = H^{(1)} \times H^{(2)}$
$G_{2(2)} \times F_{4(4)}$	$(\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R) \times (\mathrm{SU}(2)_L \times \mathrm{USp}(6)_R)$
$G_2 \times F_{4(-20)}$	$(G_2)_L \times \mathrm{SO}(9)_R$
$E_{6(6)} \times \mathrm{SL}(3)$	$\mathrm{USp}(8)_L \times \mathrm{SU}(2)_L$
$E_{6(2)} \times \mathrm{SU}(2,1)$	$(\mathrm{SU}(6)_L \times \mathrm{SU}(2)_R) \times (\mathrm{SU}(2)_R \times \mathrm{U}(1)_L)$
$E_{6(-14)} \times \mathrm{SU}(3)$	$(\mathrm{SO}(10)_L \times \mathrm{U}(1)_R) \times \mathrm{SU}(3)_R$
$E_{7(7)} \times \mathrm{SL}(2)$	$\mathrm{SU}(8)_L \times \mathrm{U}(1)_L$
$E_{7(-5)} \times \mathrm{SU}(2)$	$(\mathrm{SO}(12)_L \times \mathrm{SU}(2)_R) \times \mathrm{SU}(2)_R$
$E_{8(8)}$	$\mathrm{SO}(16)_L$

Table 2: Exceptional noncompact gauge groups and their maximal compact subgroups. The subscripts L and R refer to the AdS supergroups $G_L \times G_R$ associated to the maximally supersymmetric groundstates of these theories, see section 6.

that its maximal compact subgroup likewise factors as $H_0 = H^{(1)} \times H^{(2)}$. Denote the embedding tensor of G_0 by

$$g\Theta_{\mathcal{MN}} = g_1\, \eta^{(1)}_{\mathcal{MN}} + g_2\, \eta^{(2)}_{\mathcal{MN}} , \tag{5.20}$$

where $\eta^{(1),(2)}$ are the embedding tensors of $G^{(1),(2)}$, respectively, and assume that (5.20) satisfies (4.5). Equation (5.19) was a particular case satisfying these assumptions. Contracting (5.20) with $\eta^{\mathcal{MN}}$ yields

$$g\theta \dim E_{8(8)} = g_1 \dim G^{(1)} + g_2 \dim G^{(2)} .$$

where the l.h.s. follows from (4.3). On the other hand, contracting (5.20) with $\eta^{IJ,KL}$ over the compact part of $E_{8(8)}$ gives

$$g\theta \dim \mathrm{SO}(16) = g_1 \dim H^{(1)} + g_2 \dim H^{(2)} ,$$

where the l.h.s. here follows from (4.6) — and is a consequence of the fact that due to (4.5) the only SO(16) singlet in $\Theta_{\mathcal{MN}}$ is given by the first term in (4.3).

From the last two equations one may extract the coupling constants g_1, g_2 of the two factors of the gauge group. Their ratio is

$$\frac{g_1}{g_2} = -\frac{15 \dim G^{(2)} - 31 \dim H^{(2)}}{15 \dim G^{(1)} - 31 \dim H^{(1)}} . \tag{5.21}$$

With the gauge groups and their compact subgroups given in table 2 we then immediately obtain the ratios of coupling constants for all these groups. In particular, no degeneration occurs where this ratio would vanish or diverge. In table 1 displayed in the introduction, we have presented a list of all the noncompact admissible subgroups $G_0 \subset E_{8(8)}$, together with their coupling constant ratios. Remarkably, the ratios as determined by (5.21) come out to be independent of the particular real form for each of these exceptional noncompact groups. This suggests that the theories whose gauge groups are different real forms of the same complexified group may be related by analytic continuation, in a similar fashion as the $\mathrm{SO}(p, 8-p)$ gaugings of the $D = 4$ theory are related via $\mathrm{SO}(8, \mathbb{C})$ [35, 36, 39]. Here, the analytic continuation would have to pass through the complex group $E_8(\mathbb{C})$.

This concludes our discussion of admissible gauge groups. We note that in addition to the groups identified in this section there should also exist non-semisimple gaugings analogous to the theories constructed in [35, 36, 39, 40]. We leave their exploration and complete classification for future study.

6. Stationary points with maximal supersymmetry

The point of vanishing scalar fields, i.e. $\mathcal{V} = I$, plays a distinguished role: it is a stationary point with maximal supersymmetry for all the theories we have constructed. Recall that the condition for stationarity was already spelled out in (4.12). At $\mathcal{V} = I$, the gauge group G_0 is broken to its maximal compact subgroup H_0. For the compact gauge group (5.12), the tensor $A_2^{I\dot{A}}$ vanishes at this point, since Θ has no contribution in the noncompact directions, cf. (4.8) and (5.7). Hence, (4.12) is satisfied; the compact gauged theory has a G_0 invariant stationary point at $\mathcal{V} = I$. For the noncompact real forms (5.15), the decomposition (5.14) implies that there is no H_0-invariant tensor in the tensor product $16 \times \overline{128}$; hence, $A_2^{I\dot{A}}$ vanishes also in these theories at $\mathcal{V} = I$. The same argument works also for the exceptional noncompact gauge groups from table 1. In summary, all the three-dimensional theories we have constructed share the stationary point $\mathcal{V} = I$.

If we denote by $\nu = \dim G_0$ and $\kappa = \dim H_0$ the dimension of the gauge group and its maximal compact subgroup, respectively, the field equations (3.32) imply that for $\mathcal{V} = I$ the vector fields split into $\nu - \kappa$ massive self-dual vectors and a H_0-CS theory of κ vector fields which do not carry propagating degrees of freedom. In this way, the erstwhile topological vector fields corresponding to the noncompact directions in G_0 acquire a mass term by a Brout-Englert-Higgs like effect as observed in [41]. Dropping the massive vector fields as well as the matter fermions, the theory then reduces to a H_0-CS theory, coupled to supergravity. Since the AdS_3 (super-)gravity itself allows the formulation as a CS theory of the AdS group $\mathrm{SO}(2, 2)$ [11, 25], the resulting theory is a CS theory with connection on a superextension of $H_0 \times \mathrm{SO}(2, 2)$. We shall determine these supergroups in the following.

In order to analyze the residual supersymmetries at the stationary point $\mathcal{V} = I$ in a little more detail, we consider the Killing spinor equations, derived from (2.5), (3.13) in absence of the vector fields:

$$0 \stackrel{!}{=} \partial_\mu \epsilon^I + \frac{1}{2}\, \mathrm{i}\gamma_a \left(A_\mu{}^a\, \delta^{IJ} - 2g\, e_\mu{}^a\, A_1^{IJ} \right) \epsilon^J \,, \tag{6.1}$$

$$0 \stackrel{!}{=} A_2^{I\dot{A}}\, \epsilon^I \,. \tag{6.2}$$

Adapting the arguments of [6] to the present case, it may be shown that (6.1) in fact implies (6.2). Namely, comparing (6.1) to (2.18) we find that every solution to (6.1) corresponds to the product of an AdS_3 Killing spinor and an eigenvector ϵ_0^I of the real symmetric matrix A_1^{IJ}; the eigenvalue α_i of A_1^{IJ} is related to the AdS radius by

$$2g\,|\alpha_i| = m\,. \tag{6.3}$$

On the other hand, the Einstein field equations derived from (3.9) imply that

$$R_{\mu\nu} = 4W_0\, g_{\mu\nu}\,, \tag{6.4}$$

where W_0 is the value of the potential (3.12) at the critical point. From (2.17) we infer the relation $m^2 = 2W_0$. Given the eigenvector ϵ_0^I of A_1^{IJ} with eigenvalue α_i, we contract (3.25) with ϵ_0^I to obtain

$$\left(2\,g^2\,\alpha_i^2 - W_0\right)\epsilon_0^I = g^2\,A_2^{I\dot{A}}A_2^{J\dot{A}}\epsilon_0^J\,. \tag{6.5}$$

If α_i satisfies (6.3), this equation indeed implies (6.2). As in higher dimensions, the number of residual supersymmetries therefore corresponds to the number of eigenvalues α_i of A_1^{IJ} satisfying (6.3). Conversely, equation (6.5) shows that $A_2^{I\dot{A}} = 0$ is a sufficient condition for a maximally supersymmetric ground state: all eigenvalues of the tensor A_1^{IJ} then satisfy (6.3), splitting into $16 = n_L + n_R$ with positive and negative sign, respectively. Altogether, we have thus shown that all the theories with noncompact gauge groups from (5.15) and table 1 possess a maximally supersymmetric ground state at $\mathcal{V} = I$. This is in marked contrast to the higher-dimensional models, where several of the noncompact gaugings do not even admit any stationary points [35, 36, 39].

Not unexpectedly, the background isometries of these groundstates are superextensions of the three-dimensional AdS group SO(2,2). Since $\mathrm{SO}(2,2) = \mathrm{SU}(1,1)_L \times \mathrm{SU}(1,1)_R$ is not simple, they are in general direct products of two simple supergroups $G_L \times G_R$. Accordingly, the sixteen supersymmetry generators split into $N = (n_L, n_R)$, such that the groups $G_{L,R}$ are $n_{L,R}$ superextensions of the $\mathrm{SU}(1,1)_{L,R}$ with bosonic subgroups

$$G_{L,R} \supset H_{L,R} \times \mathrm{SU}(1,1)_{L,R}\,. \tag{6.6}$$

A list of possible factors $G_{L,R}$ based on the classification [42, 43] is given in [19].

To determine the AdS supergroups $G_L \times G_R$ corresponding to the maximally supersymmetric ground state of the theory with gauge group G_0, one must identify the groups $H_{L,R}$ among the simple factors of its maximally compact subgroup H_0, such that $H_0 = H_L \times H_R$. This basically follows from the decomposition of the sixteen supercharges under H_0. Note that H_L is not necessarily entirely contained in one of the two factors of the semisimple gauge group G_0. Rather we find that in the two factorizations

$$H^{(1)} \times H^{(2)} = H_0 = H_L \times H_R\,, \tag{6.7}$$

the subfactors are distributed in different ways among the two factors. This has been

gauge group G_0	$N = (n_L, n_R)$	background supergroup $G_L \times G_R$
$\mathrm{SO}(8) \times \mathrm{SO}(8)$	$(8,8)$	$\mathrm{OSp}(8\|2,\mathbb{R}) \times \mathrm{OSp}(8\|2,\mathbb{R})$
$\mathrm{SO}(7,1) \times \mathrm{SO}(7,1)$	$(8,8)$	$F(4) \times F(4)$
$\mathrm{SO}(6,2) \times \mathrm{SO}(6,2)$	$(8,8)$	$\mathrm{SU}(4\|1,1) \times \mathrm{SU}(4\|1,1)$
$\mathrm{SO}(5,3) \times \mathrm{SO}(5,3)$	$(8,8)$	$\mathrm{OSp}(4^*\|4) \times \mathrm{OSp}(4^*\|4)$
$\mathrm{SO}(4,4) \times \mathrm{SO}(4,4)$	$(8,8)$	$[D^1(2,1;-1) \times \mathrm{SU}(2\|2)]^2$
$G_{2(2)} \times F_{4(4)}$	$(4,12)$	$D^1(2,1;-2/3) \times \mathrm{OSp}(4^*\|6)$
$G_2 \times F_{4(-20)}$	$(7,9)$	$G(3) \times \mathrm{OSp}(9\|2,\mathbb{R})$
$E_{6(6)} \times \mathrm{SL}(3)$	$(16,0)$	$\mathrm{OSp}(4^*\|8) \times \mathrm{SU}(1,1)$
$E_{6(2)} \times \mathrm{SU}(2,1)$	$(12,4)$	$\mathrm{SU}(6\|1,1) \times D^1(2,1;-1/2)$
$E_{6(-14)} \times \mathrm{SU}(3)$	$(10,6)$	$\mathrm{OSp}(10\|2,\mathbb{R}) \times \mathrm{SU}(3\|1,1)$
$E_{7(7)} \times \mathrm{SL}(2)$	$(16,0)$	$\mathrm{SU}(8\|1,1) \times \mathrm{SU}(1,1)$
$E_{7(-5)} \times \mathrm{SU}(2)$	$(12,4)$	$\mathrm{OSp}(12\|2,\mathbb{R}) \times D^1(2,1;-1/3)$
$E_{8(8)}$	$(16,0)$	$\mathrm{OSp}(16\|2,\mathbb{R}) \times \mathrm{SU}(1,1)$

Table 3: Background isometries of the maximally supersymmetric ground states.

made explicit in (5.16) and table 2, respectively, by designating the simple factors of H_0 with the corresponding sub- and superscripts. In fact, the only gauge groups for which the two factorizations (6.7) coincide are the compact group (5.12), the group $G_2 \times F_{4(-20)}$ from table 2, and the gauge group $E_{8(8)}$ itself. For the noncompact gauge groups $E_{6(6)} \times \mathrm{SL}(3)$, $E_{7(7)} \times \mathrm{SL}(2)$, and $E_{8(8)}$, we find $H_0 = H_L$, i.e. G_R reduces to its purely bosonic AdS part $\mathrm{SU}(1,1)_R$. Another particular situation arises for the noncompact gauge group $\mathrm{SO}(4,4) \times \mathrm{SO}(4,4)$, where the supergroups $G_{L,R}$ themselves are not simple but direct products of two supergroups, respectively.

The complete list is given in table 3, where we have summarized the background isometries of the maximally supersymmetric stationary point $\mathcal{V} = I$ for all the three-dimensional gauged maximal supergravities constructed in this article.

Let us emphasize that this table presumably represents only the tip of the iceberg as we expect there to be a wealth of stationary points with partially broken supersymmetry for "small" gauge groups $G_0 \subset E_{8(8)}$. On the other hand, for "large" gauge groups stationary points will be more scarce. As a special example, consider the extremal theory with noncompact gauge group $E_{8(8)}$, for which the potential becomes just a (cosmological) constant, and does not exhibit any stationary points besides the trivial one. In this case $\mathcal{V} = I$ may always be achieved by gauge fixing the local $E_{8(8)}$ symmetry. Even after this gauge fixing, by which the scalar fields have been eliminated altogether, there still remains the "composite" local $\mathrm{SO}(16)$ invariance rendering 120 vectors out of the 248 vector fields unphysical. Accordingly, the theory in this gauge may be interpreted as an $\mathrm{SO}(16)$ Chern-Simons theory coupled to 128 massive selfdual vector fields, each of which represents one physical degree of freedom. In other words, with respect to the ungauged theory, the propagating degrees of freedom have been shifted from the scalar fields to massive selfdual vectors. This is in fact an extremal case of the mechanism required for gauging higher dimensional supergravities in odd dimensions [44, 3, 4, 6] whereby massless $k-1$ forms in a $2k+1$ dimensional space-time upon gauging turn into massive selfdual k-forms. As discussed above, truncating the massive vector fields together with the matter fermions, the theory reduces to the $\mathrm{OSp}(16|2,\mathbb{R})$ theory of [11] and reproduces its $(16,0)$ supersymmetric ground state.

It will be most interesting to study the boundary theories associated with the gauged supergravities. The background isometries given in table 3 determine the superconformal symmetries of the theories on the AdS_3 boundary. The chiral algebras are obtained by hamiltonian reduction of the current algebras based on the AdS_3 supergroups G_L and G_R, respectively (see [45] for a discussion and a translation table). For instance, the boundary theory of the superextended Chern-Simons theories [11] is described by a super-Liouville action with $\mathrm{SO}(n)$ extended superconformal symmetry [46, 47]. The maximal gauged supergravities (3.9) then introduce additional scalar and massive vector degrees of freedom, respectively, which propagate in the bulk.

7. Outlook: a higher dimensional ancestor?

As already pointed out in the introduction there appears to be no way to obtain the gauged models constructed in this paper by means of a conventional Kaluza Klein compactification, because the latter would give rise to a standard Yang-Mills-type lagrangian with a kinetic term for the vector fields, instead of the CS term that was required here. Moreover, $D=11$ supergravity does not admit maximally supersymmetric groundstates of the type $AdS_3 \times \mathcal{M}_8$ (see e.g. [48]), and even if it did, there simply are no 8-manifolds $\mathcal{M}_8$ whose isometry groups would coincide with the gauge groups G_0 that we have found (since there are no 7-manifolds with these isometries either, the arguments a fortiori also excludes type-IIB theory as a possible ancestor). Nonetheless all these gauged models constitute continuous deformations of the original $N=16$ theory of [22], which itself *is* derivable by a torus reduction of $D=11$ supergravity. The situation is therefore quite different from the one in dimensions $D \geq 4$ where the gauged theories do emerge via sphere compactifications of $D=11$ supergravity.[7] This raises the question whether there exists a higher-dimensional ancestor theory that would give rise to these theories, and if so, what it might be. While we have no answer to this question at the moment, we would like here to offer some hints.

[7] For the $AdS_4 \times S^7$ compactification this was rigorously shown in [49], while for the $AdS_7 \times S^4$ a complete proof was given more recently [50]. By contrast, the full consistency of the $AdS_5 \times S^5$ truncation of IIB supergravity remains an open problem despite much supporting evidence, see [51] and references therein.

Obviously, a crucial step in our construction was the introduction "by hand" of up to 248 vector fields $B_\mu{}^{\mathcal{M}}$ subject to the transformation rules

$$\delta B_\mu{}^{\mathcal{M}} = -2\,\mathcal{V}^{\mathcal{M}}{}_{IJ}\,\bar{\epsilon}^I \psi_\mu^I + \mathrm{i}\Gamma^I_{A\dot{A}}\,\mathcal{V}^{\mathcal{M}}{}_A\,\bar{\epsilon}^I \gamma_\mu \chi^{\dot{A}}\,.$$

As mentioned before, for the 36 vector fields associated with the 36 commuting nilpotent directions in the $E_{8(8)}$ Lie algebra, this formula can be derived directly from eleven dimensions [32]. Owing to the on-shell equivalence of vectors and scalars, vector fields can be added with impunity in three dimensions, but in extrapolating this step to eleven dimensions we seem to run into an obstacle, because extra vector fields would normally introduce new and unwanted propagating degrees of freedom. Nevertheless, the evidence for a generalized vielbein in eleven dimensions presented in [52, 53, 32], and the fact that a consistent gauging in three dimensions based on this extrapolation does exist, prompt us to conjecture that *all* 248 vector fields introduced here have an eleven-dimensional origin. In [32] it was observed that the physical bosonic degrees of freedom can be assembled into a 248-bein, which is just the lift of the $E_{8(8)}$ matrix $\mathcal{V}$ to eleven dimensions. Assuming that there are indeed 248 vector fields, all bosonic fields would thus naturally fit into a (3+248)-bein

$$\begin{pmatrix} e_\mu{}^\alpha & B_\mu{}^{\mathcal{M}} \mathcal{V}_{\mathcal{M}}{}^{\mathcal{A}} \\ 0 & \mathcal{V}_{\mathcal{M}}{}^{\mathcal{A}} \end{pmatrix},$$

which would also incorporate the three-form degrees of freedom and would replace the original elfbein of $D = 11$ supergravity

$$\begin{pmatrix} e_\mu{}^\alpha & B_\mu{}^m e_m{}^a \\ 0 & e_m{}^a \end{pmatrix},$$

The latter is just an element of the coset space $\mathrm{GL}(11,\mathbb{R})/\,\mathrm{SO}(1,10)$ in a special gauge where the tangent space symmetry is broken to $\mathrm{SO}(1,2)\times\mathrm{SO}(8)$. However, an analogous interpretation of the above (3+248)-bein remains to be found. Amongst other things, it would require replacing the action of the global $E_{8(8)}$ on the 248-bein $\mathcal{V}_{\mathcal{M}}{}^{\mathcal{A}}$ by some new type of general coordinate transformations, in the same way as GL(11) is replaced by diffeomorphisms in the vielbein description of Einstein's theory [32]. The gauge groups found in the compactification to three dimensions would then emerge as "isometry groups" in a suitable sense. We also note that for the tangent space group we have the embedding $\mathrm{SO}(1,2)\times\mathrm{SO}(16)\subset\mathrm{OSp}(32)$, but there is no simple group generalizing GL(11) that would contain $\mathrm{GL}(3)\times E_{8(8)}$ and yield the right number of (bosonic) physical degrees of freedom upon division by OSp(32) (see, however, [54] for an alternative ansatz based on the embedding $\mathrm{OSp}(32)\subset\mathrm{OSp}(64|1)$).

The challenge is therefore to find a reformulation of $D = 11$ supergravity in terms of the above (3+248)-bein and an action, which must still describe no more than 128 massless bosonic physical degrees of freedom, despite the presence of new field components in eleven dimensions. The only way to achieve this appears to be via a CS-like action in eleven dimensions that would encompass all degrees of freedom, and thus unify the Einstein-Hilbert and three-form actions of the original theory.[8] In making these speculations we are encouraged by the fact that, at least in three dimensions, the dreibein ${e_\mu}^\alpha$, the gravitinos ψ^I_μ and the vector fields are all governed by CS-type actions.

A. $E_{8(8)}$ conventions

The $E_{8(8)}$ generators $t^{\mathcal{A}}$ are split into 120 compact ones $X^{IJ} \equiv -X^{JI}$ and 128 noncompact ones Y^A, with SO(16) vector indices $I, J, \ldots \in \underline{16}$, spinor indices $A, B, \ldots \in \underline{128}$, and the collective labels $\mathcal{A}, \mathcal{B}, \ldots = ([IJ], A), \ldots$. The conjugate SO(16) spinors are labeled by dotted indices $\dot{A}, \dot{B}, \ldots$ In this SO(16) basis the totally antisymmetric $E_{8(8)}$ structure constants $f^{\mathcal{ABC}}$ possess the non-vanishing components

$$f^{IJ,\,KL,\,MN} = -8\,\delta^{\,\overline{I[K}}\,\delta^{\overline{L]J}}_{MN}\,, \qquad f^{IJ,\,A,\,B} = -\frac{1}{2}\Gamma^{IJ}_{AB}\,. \tag{A.1}$$

$E_{8(8)}$ indices are raised and lowered by means of the Cartan-Killing metric

$$\eta^{\mathcal{AB}} = \frac{1}{60}\,\mathrm{Tr}\, t^{\mathcal{A}} t^{\mathcal{B}} = -\frac{1}{60} {f^{\mathcal{A}}}_{\mathcal{CD}} f^{\mathcal{BCD}}\,, \tag{A.2}$$

with components $\eta^{AB} = \delta^{AB}$ and $\eta^{IJ\,KL} = -2\delta^{IJ}_{KL}$. When summing over antisymmetrized index pairs $[IJ]$, an extra factor of $1/2$ is always understood. Explicitly, the commutators are

$$\begin{aligned}
[X^{IJ}, X^{KL}] &= 4\,\delta^{\,\overline{I[K}}X^{\overline{L]J}}\,, \\
[X^{IJ}, Y^A] &= -\frac{1}{2}\Gamma^{IJ}_{AB} Y^B\,, \\
[Y^A, Y^B] &= \frac{1}{4}\Gamma^{IJ}_{AB} X^{IJ}\,.
\end{aligned} \tag{A.3}$$

The equivalence of the fundamental and the adjoint representations of $E_{8(8)}$ plays an important role in our considerations; it is expressed by the relation

$$\mathcal{V}^{-1} t^{\mathcal{M}}\, \mathcal{V} = {\mathcal{V}^{\mathcal{M}}}_{\mathcal{A}}\, t^{\mathcal{A}} \qquad \Longleftrightarrow \qquad {\mathcal{V}^{\mathcal{M}}}_{\mathcal{A}} = \frac{1}{60}\,\mathrm{Tr}\,(t^{\mathcal{M}}\,\mathcal{V}\, t_{\mathcal{A}}\, \mathcal{V}^{-1})\,. \tag{A.4}$$

Further formulas concerning the $E_{8(8)}$ Lie algebra, which will be used in this paper can be found in [34, 32].

Let us finally point out that in the main text we use collective labels $\mathcal{A}, \mathcal{B}, \ldots$ and $\mathcal{M}, \mathcal{N}, \ldots$ for the $E_{8(8)}$ matrix ${\mathcal{V}^{\mathcal{M}}}_{\mathcal{A}}$ defined in (A.4), to distinguish the transformation of these indices under the left and right action of $E_{8(8)}$ and SO(16), respectively, according to (2.1). Likewise, $\Theta_{\mathcal{MN}}$ is an $E_{8(8)}$ tensor whereas $T_{\mathcal{A}|\mathcal{B}}$ transforms under the local SO(16), cf. (3.21).

[8] We are aware that the idea of reformulating $D = 11$ supergravity as a CS theory is not entirely new. However, the present ansatz is evidently very different from previous attempts in this direction.

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